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MODELLING CONTAGION AND INTEGRATION EFFECTS ON RISK MANAGEMENT MEASURES♣

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Abstract

We add to the risk management debate concerning extensions of Value-at-Risk (VaR), following a research route that relaxes the unreliable statistical assumptions. We propose an innovative VaR measure based on time-varying moments of a best fitting distribution extracted using multivariate ARMA-GARCH. We provide a VaR that is able to capture the cross-effects associated with contagion and integration. This refined statistical risk metric is useful for samples of assets where the influence of common risk drivers should not be negligible. We implement an empirical exercise applying Basel VaR and our VaR, with and without the cross-effects in a sample of the main worldwide financial sector indices of G20 economies, covering a period sufficiently extensive. According to Basel backtesting, we reject Basel VaR in all economies and univariate VaR in four cases. The multivariate VaR is rejected in only one case: ASX 200 Financials in Australia. According to backtesting that deals with the frequency and conditionality of losses exceeding VaR, while Basel VaR is rejected for all ten economies, we fail to reject our multivariate VaR for all assets. Except for IFNC index in Brazil, our multivariate VaR shows the best performance according to the average violation and Lopez (1999) ranking criteria.

Keywords: Value at Risk; Time-varying moments; Laplace and Hyperbolic secant distributions; Cross-effects.

JEL Codes: G15; C32

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1 INTRODUCTION

According to Bernstein (1996), the use of statistical frameworks aiming an active role in risk management in various financial transactions can be seen as one of the landmarks able to characterize the evolution of society over time, as well as technology, capitalism and democratic maturity.

This strand of the financial literature depends basically on applying probability theory, with emphasis on families of continuous time probability distribution functions (pdf), as the normal suggested by De Moivre (1738).

However, despite its relevance and essence, risk management is not a classic area of research in finance. The most relevant related contributions are recent when compared to the asset pricing models following a random walk developed by Cardano (1565), or to demographic studies facing actuary purposes developed by the British mathematicians since the eighteenth century.

In this historical scenario, we can characterize the financial market as seemingly slow and passive in the face of macroeconomic influence on risk management before the 70's. This is an understandable behavior, because of the long periods of stability and high levels of predictability during this period. We can observe a more active signal of the market dealing with risk management in a more standardized way only during the 70’s and 80’s. Those decades were marked by disasters, as the loss of about US$300 million in 1982 reported by Chase Manhattan Bank.

After these damage reports, in 1988 more specifically, international agencies seemed to converge aiming to establish regulatory frameworks, norms and mechanisms to manage banking risks. In concrete terms, there was the implementation of the agreements reached by the Basel Committee, in 1992, and the Basel III agreement currently in implementation.

Thenceforth, we can evidence a growing concern in the improvement of these issues and in this context, we necessarily need to deal with the metrics used in the measurement and management of various types of risk inherent to financial system. More specifically, risk management measures can affect stabilization through reducing the impact of fluctuations on the treasuries of the financial system institutions.

The literature used to associate the modern risk management theory to the report prepared by J. P. Morgan, and more specifically to the concept established by this institution in 1994 labeled Value at Risk, or VaR. This metric takes into account the need for dynamic, uniform and objective risk metric due to more frequent turbulent scenarios. Thus, VaR emerges as universal metric of risk measurement, first because of the prestige of J. P. Morgan.

Second, we must mention the characteristics of this metric, not necessarily in a technical sense as suggested in Artzner et al. (1999), but in terms of the conceptual sense since it accounts for some of the desired rationales: VaR works as a partial moment of the distribution associated with extreme loss.

Compared to more refined risk metrics, as the drawdown that handles the accumulated maximum loss, VaR aims at a loss associated with extremely negative scenarios. VaR captures an overall risk extreme and not a systemic risk as measured by the market beta derived from the Capital Asset Pricing Model (CAPM).

VaR also does not have the property of being a metric relative to a benchmark, as well as the tracking error volatility, commonly used in investment fund management. VaR only takes into account the moments of the probability distribution function of the asset in question.

In its simplest nonparametric version, VaR is a negative result that occurs with a certain cumulative probability from a histogram. It is also simple to measure a VaR by the means of a historical simulation.

In its most commonly used versions, this metric depends on a parametric statistical framework based on some unreliable assumptions. The traditional unconditional Gaussian VaR refers to the premise that one should not reject the null hypothesis that the net return on financial assets follows a normal pdf, with moments fixed over time that depends only on the time series of the return on own asset.

According to Jorion (2007), although it is common to assume the Gaussianity of stock returns, this pdf does not accommodate patterns of asymmetry nor leptokurtosis, aspects usually evidenced in financial markets, as we can see in the extensive literature, since Levy (1925), Mandelbrot (1963) and Fama (1965).

Thus, an important step in order to manage risk is to infer about the suitability or not of unconditional normality distribution, by the means of Jarque and Bera (1981) test, for instance, and then dealing with this violation by seeking the most appropriate fitting function.

Other step is to deal with the issue of conditional moments, specifically, the mean and standard deviation useful to measure VaR, by extracting both as time-varying series from some statistical approach, instead of constant parameters.

As a final step, we incorporate cross-effects on the extraction of the time series of both moments aiming to deal with the effects of integration and financial contagion reported for several samples of economies in Fasolo (2006), Chuang Lu and Tswei (2007), Beirne and Giek (2012), Matos, Oquendo and Trompieri (2013) and Puig and Rivero (2014), for instance. The main question that motivates us here is whether the current risk measurement capture these cross-effects, thus providing good predictions. In other words, we intend to know if the cross-effects are of second order or not.

We rely on a statistically sophisticated approach taking into account some of the main criticisms of traditional parametric measures of VaR, by proposing a metric which depends on time-varying moments of a best fitting distribution, derived to be able to capture the cross-effects associated with common risk drivers.

We apply our methodology in a sample of main worldwide financial sector indices, which is comprised by the banking, insurance and financial intermediation companies. In our empirical exercise, we collect financial sector index data for G20 economies, covering a period seen as sufficiently extensive: at least one thousand daily observations.

Our final cross-section is composed by financial sector indices of Australia, Brazil, Canada, Germany, France, India, Mexico, UK, USA and Russia, covering the period from March 30, 2009 to December 31, 2013, 1255 observations. The validation of all types of VaR used here is given by using the following backtesting methods: Basel, Lopez (1999), Kupiec (1995) and Christoffersen (1998) and the joint test proposed by these authors.

Our main findings corroborate previous evidences reported by Berkowitz and O'Brien (2002) for large US banks: the potential concern due to cross-effects.

Our VaR measure performs better than Basel and also univariate versions based on most backtesting methods. Based on Basel violation test we fail to reject our VaR for all economies, except for ASX 200 Financial Index in Australia. We reject Basel VaR for all economies, while Univariate model is rejected for four economies. Our multivariate approach also performs better when we use other backtesting methods.

We believe that this evidence may motivate the literature to propose statistical improvements to future versions of Basel VaR.

The work is structured as the following. Section 2 describes the methodology. The empirical exercise is reported in section 3. The final considerations are in the fourth section.

2 METHODOLOGY

2.1. An Overview of the Related Literature

In 1994, RiskMetrics defined VaR as a single and simple metric of risk which assumes that returns follow a normal distribution and that volatility follows an exponential smoothing process rather than a standard deviation.[4] This is a milestone in the recent risk management literature, because VaR has influenced financial system procedures. For instance, Basel Committee uses VaR as a legal framework on signatory countries.

The question however, inherent in the evolution of science is the need to better accommodate the basic violations usually evidenced for series of returns on assets.

[4] EWMA is a term used by RiskMetrics referring to Exponential Weighted Moving Average. See Laubsch and Ulmer (1999) for more details.

Regarding the moments of distribution, West and Cho (1995) have shown that for short time horizons, models following the family of frameworks untitled Generalized Autoregressive Heteroskedasticity (GARCH) – originally developed by Engle (1982) and generalized by Bollerslev (1986) – are more accurate and suitable for predicting volatility, than simply the constant standard deviation or even compared to other frameworks of conditional volatility. In this path, Danielssson and de Vries (1998) give a first step, showing the importance of incorporating it to GARCH–VaR.

Another relevant step in this specific literature is given by Lee and Lee (2009) and Rippel and Jánský (2011). These almost parallel studies innovate using Autoregressive Moving Average (ARMA) to model asset return levels and the GARCH for volatility; thus creating the family ARMA–GARCH VaR.

These extensions are very relevant for literature, however they do not care about the inadequacy of the pdf. In short, Jorion (2007) suggests using standard parametric distributions. Even assuming that returns are independent and identically distributed (IID) in any of its versions, strong or weak, the consensual evidence reported in this literature suggest that this pdf is not normal. See Mandelbrot (1963) and Fama (1965), as pioneering references.

In this sense, we need to address this issue about pdf by incorporating statistical properties as time-varying moments to a better specified non-normal distributions. In Hull and White (1998), Vaughan (2002) and Klein and Fischer (2003), we can see the benefits of using other distributions, as Generalized Secant Hyperbolic distribution.

In this promising route, Matos et al. (2015) suggest an important methodological contribution, but under a univariate context, in which contagion effects and financial integration between the markets are assumed to be of second order.

A final contribution that motivates us to propose an innovation to this literature is reported in Cappielo, Engle and Sheppard (2006). They suggest an improvement to previous approaches due to the inclusion of cross-effects by means the estimation of a multivariate GARCH.

Our contribution to this debate is to propose an innovative VaR entitled Multivariate Autoregressive Moving Average – Generalized Autoregressive Conditional Heterocedasticity – Best Fitting Value at Risk. More simply, we derive a VaR, denoted by VaR^{MBFC}, based on the best fitting pdf, whose time-varying moments follow a multivariate ARMA–GARCH framework.

In the next subsection we give the most relevant details of this conditional measure of risk.

2.2. Our VaR^{MBFC} in Details

Taking as an example the Gaussian pdf, the relationship of unconditional VaR at a given confidence level $\mathrm{c}(\%)$ expressed by $\mathrm{VaR}^{\mathrm{GU}}(\mathrm{c}\%)$, is given by:

$$VaR^{GU}(c\%) = \mu - \sigma\alpha_{c(\%)} \tag{1}$$

where μ is the population mean parameter, σ is the parameter that measures the population standard-deviation and $\alpha_{c(\%)}$ is the characteristic alpha level in a normal standard, which takes the value of 2.32630 for a cumulative probability of 1 % and 1.64485 for a cumulative probability of 5 %, for instance.

This relationship is the quantile function of a gaussian pdf, i.e., the inverse of the cumulative distribution function associated with a one-tailed probability, 5% or 1%, which is related to the confidence level, $c(\%)$, according to the relationship given by $1 - c(\%)$.

The first main issue to be addressed here is how to derive an extension of this unconditional approach, but taking into account another pdf instead of the normal one. In this sense, our Best-fitting Unconditional VaR, is given by:

$$VaR^{BFU}(c\%) = F_{BF}^{-1}(1 - c|\boldsymbol{\Theta}) \tag{2}$$

where Θ is the coefficient vector of the respective pdf and F_{BF}^{-1} means the inverse of cumulative of this same pdf, i.e., its quantile function.

Unfortunately, the search for this specific and idiosyncratic distribution needs to impose a limitation on the range of continuous distribution families, because we can only use pdf´s in which the standard deviation and the mean are given by univariate bijections, i.e., each moment depends on only one pdf parameter.

Therefore, we have to find the quantile relationship, in which one can make the assumption that certain parameters of the distribution are time-varying so that it can accommodate the evidence that the mean and volatility are both conditionally time-varying.

Observe that the inclusion of time-varying moments in a gaussian pdf is straightforward, since the mean and standard deviation, both extracted using ARMA-GARCH, given respectively by μ_t and σ_t, will replace the respective constant parameters. [5] So, a Gaussian ARMA–GARCH VaR expressed by $VaR^{GC}(c\%)$, is given by:

$$VaR^{GC}(c\%) = \mu_t - \alpha_{c(\%)}\sigma_t \tag{3}$$

Our assumption is that we need to identify exactly which parameter is time-varying so that the average also moves, and the same applies for the standard deviation formula. Otherwise, the evidence that the average and the deviation are both time-varying do not have

[5] Aiming to estimate the multivariate GARCH here, we follow Cappielo, Engle and Sheppard (2006) by using Asymmetric Generalized Dynamic Conditional Correlation (AGDCC) specification due to its ability to capture cross-correlations between financial assets dynamically over time, besides accepting that their innovations are asymmetrical between assets.

exact counterpart on the assumption that the distribution parameters are also exactly time-varying.

In other words, assuming that the average is conditional, but it depends on two or more parameters of the distribution in question, how can we implement the bijection to replace the parameter by the average in the formula of the quantile function?

For example, in Dagun (4p) function, a pdf with good fitting for Brazilian banks returns, the standard deviation function has as arguments all 4 parameters of the distribution, which also appear in the quantile function. So, it seems impossible to establish a relation of this inverse accumulated and the fixed standard deviation to be replaced by the conditional standard deviation.

Therefore, it is necessary to obtain a bijection, such that these parameters become a function of average and deviation, respectively, so that the inverse cumulative function, which depends on specific pdf parameters, can be expressed by the average and standard deviation, which will be considered as time-varying.

Finally, the incorporation of conditional moments series extracted from a Multivariate ARMA–GARCH into the quantile function of the best fitting pdf allows us to propose an innovative VaR, denoted by $VaR^{MBFC}(c\%)$ and given by:

$$\mathrm{VaR}^{\mathrm{MBFC}}(\mathrm{c}\%) = \mathrm{F}_{\mathrm{BF}}^{-1}(1 - \mathrm{c}|\mu_{\mathrm{t}}, \sigma_{\mathrm{t}}) \tag{4}$$

Specifically on the probability distributions, the ranking in terms of fitting is prepared based on the adhesion test of Anderson and Darling (1952), following Prause (1999), where this test was used to adjust distributions to German banks. This test is a more sensitive variation of the Kolmogorov-Smirnov test, and therefore more suitable for heavy-tailed distributions. [6]

According to our results reported in the next section, we have for some economies the Laplace as the best fitting pdf and for other economies the hyperbolic secant distribution as the best one.

For the probability of Laplace distribution function whose parameters are μ and λ, and whose standard deviation is given by $\sigma = \sqrt{2}/\lambda$, it is straightforward the adaptation of the relation (4).

Laplace based Multivariate ARMA–GARCH, denoted by VaR^{MLapC} is given by:

$$\mathrm{VaR}^{\mathrm{MLapC}}(\mathrm{c}\%) = \mu_{\mathrm{t}} + \sigma_{\mathrm{t}} \frac{\ln(2.(1 - \mathrm{c}\%))}{\sqrt{2}} \tag{5}$$

[6] This test was developed by Kolmogorov (1933) and Smirnov (1933).

In the case of economies whose indices follow a hyperbolic secant distribution, the mean and standard deviation are given directly by μ and σ parameters. So, it is straightforward the adaptation of the relation (4) for this distribution.

Hyperbolic secant based Multivariate ARMA–GARCH, denoted by VaR^{MHSC} is given by:

$$\mathrm{VaR}^{\mathrm{MHSC}}(c\%) = \mu_t + 2.\sigma_t \frac{\ln\left\{\mathrm{tang}\left[\frac{\pi(1-c\%)}{2}\right]\right\}}{\pi} \tag{6}$$

2.3. Backtesting

As usual in the literature on VaR extensions, for each VaR specification used here, predictions are made one day forward for a confidence level of 99%. To compare the specifications we use backtesting methods of Basel, Lopez (1999), Kupiec (1995), Christoffersen (1998) and the joint test. The reason for using all these tests is due to the specific features of each one. [7]

More specifically, the pattern by the Basel agreement is based on a number of VaR violations. We may reject a VaR measure whether the number of violations is greater than expected. In the test proposed Lopez (1999), the purpose is to establish a ranking of the models from the measurement of the size of the losses by means the loss of function, without using formal statistical able to reject or not a VaR model. In this same sense, we also measure two partial statistics useful to rank two or more metrics of VaR. We measure the excess conservatism and average violation using the same formula of semivariance, for instance, however taking into account the values of VaR and of the respective return. In the first measure, we consider only the deviations when there is no violation and the second measure is associated with violations.

The Kupiec test (1995) is based on frequency losses exceeding VaR, in order to verify statistically the frequency of loss model is consistent with the expected statistical distribution. In those tests described, we do not consider any information about the size of violations or if they have cluster patterns. Aiming to accommodate this, Christoffersen (1998) developed a test aiming to deal with the conditionality of losses exceeding VaR, which is expected to be unconditional. The test suggested by Kupiec and Christoffersen simultaneously analyzes the frequency and conditionality of losses exceeding the VaR, allowing you to check if the VaR excess losses have the correct frequency and are unconditional or follow a statistical distribution.

[7] See Campbell (2005) for a comprehensive and comparative study of backtesting specifications. See Pena Rivera & Ruiz -Mata (2006) for other proposals for measuring the quality of risk metrics.

3. EMPIRICAL EXERCISE

3.1. Data and Summary Statistics

Major market indices are well known and they use to be composed of stocks of companies from several sectors of the economy. However, by incorporating all these companies, we lose the power of explaining a particular segment of the market. Aiming to deal with this issue, we observe the appearing of sectorial indices, with a proposal to complement the general market indices and also providing summary information about a specific sector of the economy, such as financial, trade, energy, consumption, among others. A recent contribution about Brazilian sectorial indices is Matos, Sampaio and De Castro (2016).

Our statistical refined risk metric is useful for samples of assets where the influence of common risk drivers should not be negligible. So, we implement an empirical exercise applying Basel VaR and our VaR considering or not the cross-effects in a sample of main worldwide financial sector indices of G20 economies. This sector consists of banks, insurance companies and other financial intermediation companies. The choice for this sector follows Longin and Solnik (1995, 2001).

To summarize, this sector has idiosyncrasies that make it more likely to be influenced by contagion and integration, because banks, insurance companies and other financial companies in major economies are often strongly connected, with interdependence in the short and long term.

In principle, whenever econometric or statistical tests are performed, it is preferable to employ a large data set either in the time-series (T) or in the cross-sectional dimension (N).

When working with worldwide financial sector indices, we have to deal with the trade-off between T and N. So, in terms of sample size, the main limitation for the time-series span used here is the appearing of this sectorial index across countries and the availability of a time series sufficiently extensive, at least one thousand daily observations, i.e., approximately four years.

Given this context, our sample consists of daily returns on financial index of ten of the most relevant economies during the period from March 30, 2009 to December 31, 2013, with a total of 1255 daily observations. In order to have a balanced database, we adjusted the data series to make these calendars uniform, since the countries have different calendars in terms of working days. The criterion is to use any day that was a working day in any of the economies.

We report in Table 1 a basic description of all financial indices.

Table 1- Description of main worldwide financial sector indices

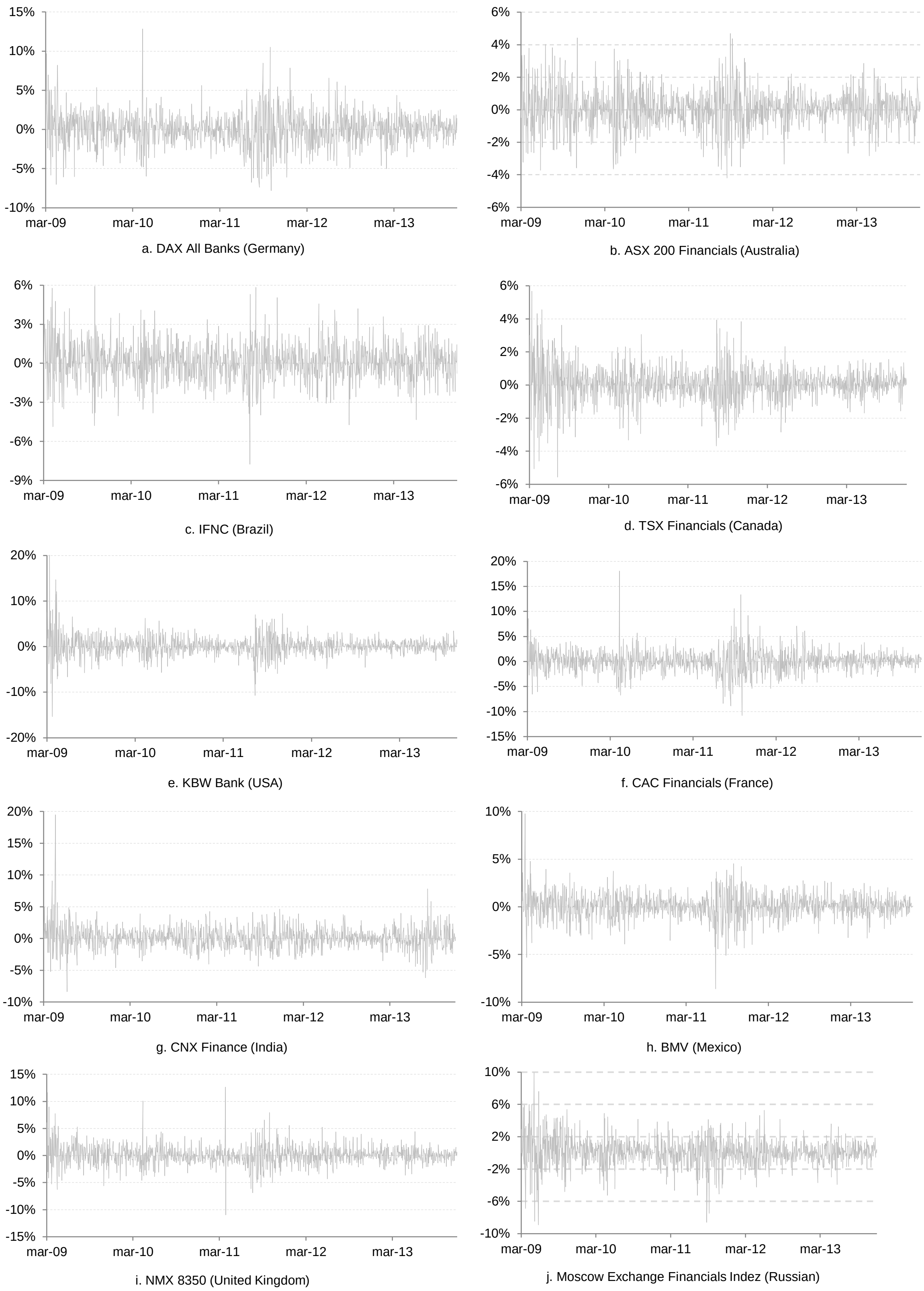
Country	Index (Financial Sector)	Continent	Position in the ranking (GDP, 2013)
Germany	DAX All Banks	Europe	4th
Australia	ASX 200 Financials	Oceania	12th
Brazil	IFNC	South America	7th
Canada	TSX Financials	North America	11th
United States of America	KBW Bank	North America	1st
France	CAC financials	Europe	5th
India	CNX Finance	Asia	10th
Mexico	BMV	North America	15th
United Kingdom	NMX 8350	Europe	6th
Russia	Moscow Exchange Financials Index	Europe	9th

We can observe indices of the financial sectors of countries located on five continents, with a greater presence of European and North American countries. Unfortunately, the relevant financial markets, such as Japanese or Chinese, or do not provide time series of its respective sectorial indices, or these series are very recent.

Figure 1 shows nominal net return on financial sector indices in terms of the local investor´s currency, based on the daily time series for the end-of-day quote. We can highlight volatility clusters and higher oscillations, mainly between 2011 and 2012, a period characterized by the sovereign debt crisis in some European countries.

Table 2 reports summary statistics and some useful statistical tests associated with violations of traditional Gaussian model for the series shown in Figure 1.

Figure 1 - Nominal net returns on main worldwide financial sector indices [a,b]



[a] This figure plots the nominal net return on financial sector index in terms of the local investor´s currency, based on the daily time series for the end-of-day quote, during the period from March 30, 2009 to December 31, 2013.
[b] Data source: Bloomberg.

Table 2 - Summary statistics and violation tests applied to returns on main worldwide financial sector indices [a]

Metrics/Index	CNX Finance (India)	DAX All Bankx (Germany)	KBW Bank (USA)	IFNC (Brazil)	ASX 200 Financials (Australia)	CAC Financials (France)	BMV (Mexico)	NMX 8350 (United Kingdom)	TSX Financials (Canada)	Moscow Exchange Financial Index (Russia)
Panel a. Summary statistics										
Gain										
Mean	0.099%	0.040%	0.100%	0.068%	0.052%	0.075%	0.071%	0.065%	0.062%	0.090%
Minimum	-8.351%	-7.807%	-15.354%	-7.762%	-4.195%	-10.776%	-8.604%	-10.959%	-5.571%	-8.904%
Maximum	19.491%	12.859%	20.107%	5.920%	4.686%	18.073%	9.760%	12.620%	5.676%	9.918%
Cumulative	189.568%	26.288%	164.755%	107.037%	77.167%	91.304%	121.279%	84.289%	104.275%	156.607%
Risk										
Standard deviation	1.701%	2.091%	2.116%	1.427%	1.165%	2.158%	1.279%	1.818%	1.033%	1.700%
Semivariance	1.119%	1.455%	1.428%	1.000%	0.822%	1.488%	0.920%	1.224%	0.734%	1.207%
Drawdown	37.596%	64.268%	41.933%	32.094%	30.582%	53.983%	33.881%	43.546%	22.050%	47.426%
3rd e 4th moments										
Asymmetry	1.337	0.263	0.790	0.045	0.037	0.551	-0.110	0.593	-0.048	-0.123
Kurtosis	17.818	5.914	15.827	4.831	4.329	9.726	8.211	9.032	6.653	7.122
Panel b. Violation tests										
Normal distribution (Jarque and Bera test) [b]										
Statistic	11901.280 ***	4615.530 ***	8768.740 ***	177.140 ***	93.610 ***	2440.530 ***	1429.540 ***	1986.080 ***	702.500 ***	896.710 ***
p-value	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Stationarity (ADF test) [c]										
Statistic	-11.086 ***	-10.725 ***	-12.009 ***	-11.585 ***	-10.443 ***	-10.973 ***	-11.009 ***	-10.842 ***	-11.263 ***	-10.206 ***
p-value	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Heteroskedasticity (ARCH LM test) [d]										
Statistic	2.826 *	13.123 ***	5.980 **	46.015 ***	29.628 ***	19.846 ***	4.493 **	68.444 ***	49.245 ***	17.645 ***
p-value	[0.093]	[0.000]	[0.015]	[0.000]	[0.000]	[0.000]	[0.034]	[0.000]	[0.000]	[0.000]

[a] Statistics of daily series of returns on returns on main worldwide financial sector indices, during the period from March 30, 2009 to December 31, 2013. Data source: Bloomberg. [b] Jarque and Bera (1981) test is performed to infer about normal distribution of returns, whose null hypothesis is normality. [c] Dickey and Fuller (1979) test in its augmented version is performed to infer about stationarity of returns, whose null hypothesis is nonstationarity. [d] ARCH LM test is performed to infer about heteroskedasticity of returns, whose null hypothesis is homoskedasticity. *** indicates rejection of null hypothesis at 1% level. ** indicates rejection of null hypothesis at 5% level. * indicates rejection of null hypothesis at 10% level.

The country whose financial index has the highest cumulative gain in the period was India, with 189%. This index does not seem to be influenced by the European sovereign credit crisis, while Germany has suffered great impact with this crisis and had the worst cumulative growth with only 26% and the largest drawdown among these indices, with 64% of highest cumulative loss. The Canadian financial sector index was less volatile considering all measures used here. Its drawdown was only 22%.

Most indices showed right asymmetry, excepted for Mexico, Canada and Russia. According to Table 2, all series are leptokurtic with a higher intensity for India and lower for Australia, an evidence that suggests the frequency of occurring large losses. These skewness and kurtosis are a strong evidence that the series does not follow a normal distribution. We corroborate this by applying the normality test developed by Jarque and Bera (1981). The result suggest the rejection of the null hypothesis of normality at the 1% significance level for all indices.

Non-stationary series may suggest explosive moments, which do not satisfy the necessary conditions for estimation of some models. Aiming to evidence the stationary or not of these time series, we perform the unit root test proposed by Dickey and Fuller (1979), in its augmented version, known as ADF test. The results of this test, at the level of significance of 1%, suggest the rejection of null hypothesis for financial series of all countries, i.e., there is no unit root.

Another common problem in financial series is the presence of heteroscedasticity. To verify whether these series are homoscedastic or not, we use the ARCH–LM test proposed by Engle (1982). We see that at 1% level of significance, eight series reject the null hypothesis of homoscedasticity, and at 10% of significance level, no series seem to be homoscedastic.

3.2 Best Fitting Probability Distribution Functions

Given rejection of the hypothesis of normality, following our procedure suggested here, we may rank probability distribution functions considering a range with over 60 statistical distributions, based on fitting measures, as the metric proposed by Anderson and Darling (1952).

According to Table 3, among more than 60 continuous distributions, the normal distribution took place between 9th and 19th. Some of the best ranked distributions are Johnson SU, Error, Hyperbolic Secant and Laplace.

However, among the subset of distributions that can establish the bijection necessary for the quantile function based on time-varying moments, the functions that appear more suitable fitting are Hyperbolic Secant and Laplace. Those distributions took place between 1st and 3rd. This table also contains the estimates of the parameters of the distributions.

The VaR Best Fitting Unconditional with 99% of confidence level, $VaR^{BFU}(99\%)$, ranges from a maximum expected daily loss of 2.9% for the Canadian financial market to almost 5.9% to the German case.

3.3 ARMA-Multivariate GARCH Models

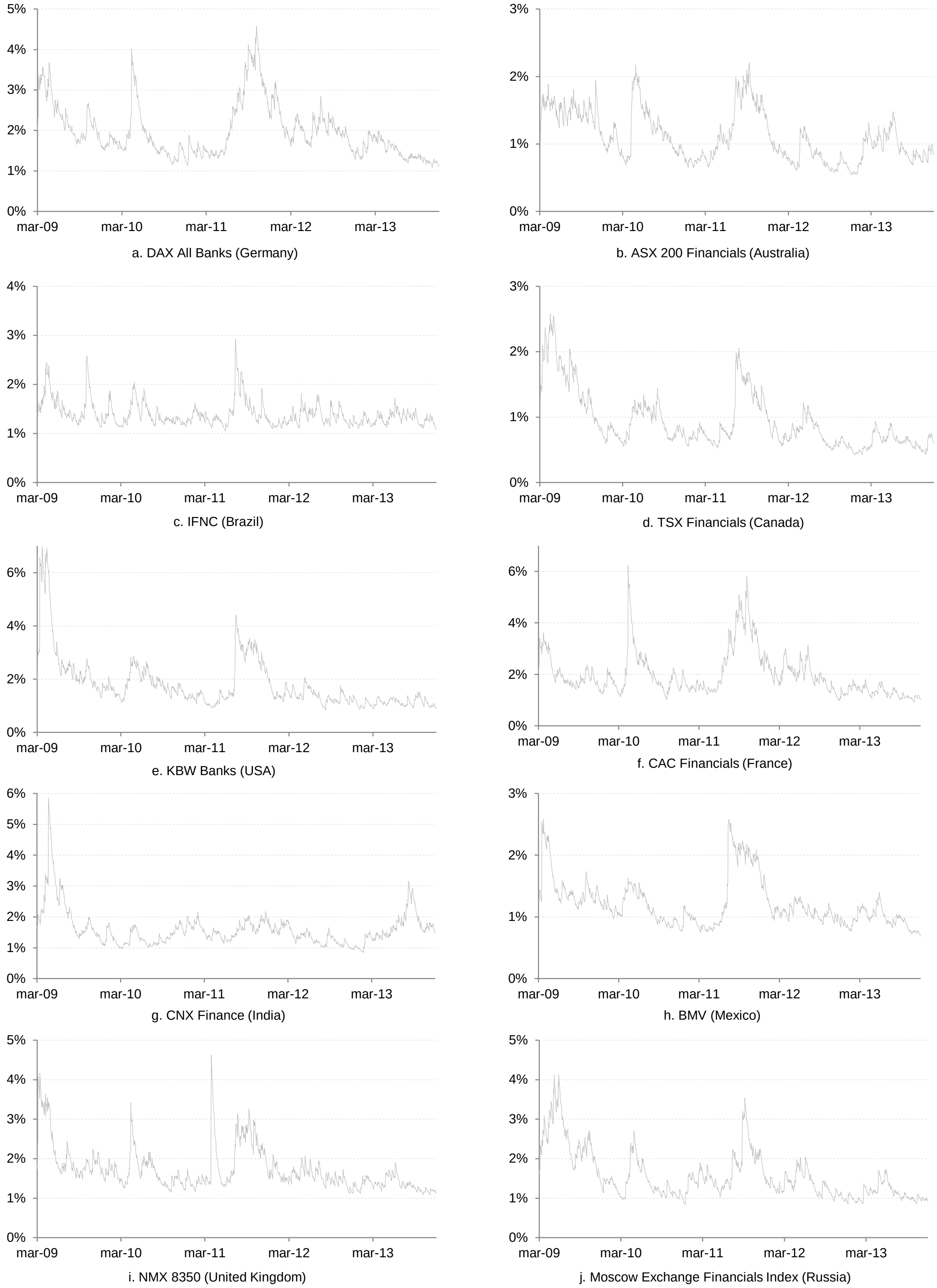
Figure 2 shows the behavior of the extracted conditional volatilities using a parsimonious multivariate ARMA–GARCH model. Analyzing visually, there seems to be clusters of volatility during turbulent periods. All series have volatile peaks in the second half of 2011, except for India whose volatility remained stable during this period. In the series of the United States index, it is clear remnants effect of the subprime crisis in the first half of 2009, with high volatility in this period, as well as in Canada and Mexico. European countries' volatile peaks are in the first half of 2010, maybe due to the first signs of the sovereign debt crisis on the continent, also demonstrating the tight integration between them.

Table 3 - Identification of the best fitting probability distribution function

Index (Financial sector)	Country	Best fitting probability distribution function (pdf) [a]	Anderson-Darling statistic test	pdf parameters	Best fitting pdf critical value (1%)	Global ranking (best fitting pdf) [b]	Global ranking (normal distribution) [b]
DAX All Banks	Germany	Hiperbolic secant	0.582	s= 0.02091; μ=0.00040	-5.489%	1st	9th
ASX 200 Financials	Australia	Hiperbolic secant	0.492	s= 0.01165 ; μ=0.00052	-3.029%	1st	11th
IFNC	Brazil	Hiperbolic secant	1.113	s= 0.01427 ; μ=0.00068	-3.704%	2nd	12th
TSX Financials	Canada	Laplace	0.778	l= 136.920 ; μ=0.00062	-2.795%	1st	12th
KBW Bank	United States of America	Laplace	3.568	l= 66.830 ; μ=0.00100	-5.754%	2nd	18th
CAC Financials	France	Laplace	0.407	l= 65.540 ; μ=0.00075	-5.894%	2nd	19th
CNX Finance	India	Laplace	1.151	l= 83.160 ; μ=0.00099	-4.605%	2nd	19th
BMV	Mexico	Laplace	0.919	l= 110.590 ; μ=0.00071	-3.466%	3rd	15th
NMX 8350	United Kingdom	Laplace	1.277	l= 77.800 ; μ=0.00065	-4.964%	1st	19th
Moscow Exchange Financials Index	Russia	Laplace	0.932	l= 83.210 ; μ=0.00090	-4.612%	1st	11th

[a] The best-fitting pdf is identified and the ranking is done based on Anderson and Darling (1952) statistic. Our search for this specific and idiosyncratic distribution needs to impose a limitation on the range of continuous distribution families, because we can only use pdf´s in which the standard deviation and the mean are given by univariate bijections, i.e., each moment depends on only one pdf parameter.
[b] This is an unrestricted ranking, considering all continuous timing distributions.

Figure 2 - Conditional volatility of returns on main worldwide financial sector indices [a,b]



[a] This figure plots the conditional volatility of nominal net return on each financial sector index, during the period from March 30, 2009 to December 31, 2013. [b] Series extracted from the estimation of a Multivariate ARMA–GARCH, following an Asymmetric Generalized Dynamic Conditional Correlation (AGDCC).

3.4. VaR Estimation and Backtesting

Aiming to suggest an innovation to Basel Committee, in Figure 3 we plot time evolution of the VaR series generated following Basel, VaR^{Bas}, and using our multivariate metric, VaR^{MBFC}, both with a confidence level of 99% and horizon of 1 day, as well as their daily return series for each banking indices. Visual analysis allows us to suggest that for all series, except the American and Mexican cases, the maximum expected losses predicted by both frameworks are apparently close.

In general, there are three moments where the highest values of VaR are perceived in almost all series, matching the times of greater volatility: i) the first half of 2009, still reflecting subprime crisis in the United States, ii) first half of 2010, showing time of instability in the euro zone due to the first signs of the sovereign debt crisis and iii) the second half of 2011, with the emergence of the same crisis signaling the possibility of some government defaults, such as Spain and Italy.

When our purpose is to infer about the relevance of contagion and financial integration effects between the banking systems of these economies, VaR^{MBFC}, which incorporates these cross-effects is compared with the VaR^{UBFC}. They are similar in all aspects, except for conditional moments incorporated into distribution with better fitting, which in the latter VaR are estimated from a univariate ARMA–GARCH, instead of multivariate.

To continue this comparison we must use the backtesting methods defined in the previous section. The results of the proposed backtestings are shown in Table 4.

According to Basel baktesting, which takes into account violations in absolute or relative amounts over the 1255 daily observations, we reject Basel VaR for all economies, while univariate VaR is rejected for four economies: Australia, Canada, USA and Mexico. Multivariate VaR is rejected in only one economy, when we measure risk management for ASX 200 Financials in Australia.

According to backtesting which deal with the frequency and conditionality of losses exceeding VaR, i.e., Kupiec (1995), Christoffersen (1998) or the joint test proposed by these authors, while VaR^{Bas} is rejected for all ten economies, we fail to reject our VaR^{MBFC} for all assets.

The univariate VaR is rejected only for Australia index. Since for most indices, there are no successive violations when we use univariate or multivariate VaR measures, we can not measure a value for the statistic test proposed by Christoffersen (1998) neither for the joint test.

Table 4 also reports useful partial metrics to measure the average violation and excessive conservatism. Except for IFNC index in Brazil, our multivariate VaR shows the best performance regarding average violation and also if we observe Lopez (1999) ranking. Based on these methods, the difference between Basel VaR and other is very high.

Multivariate VaR seems to be the most conservative of all considering all economies.

Figura 3 - VaR of returns on main worldwide financial sector indices [a]

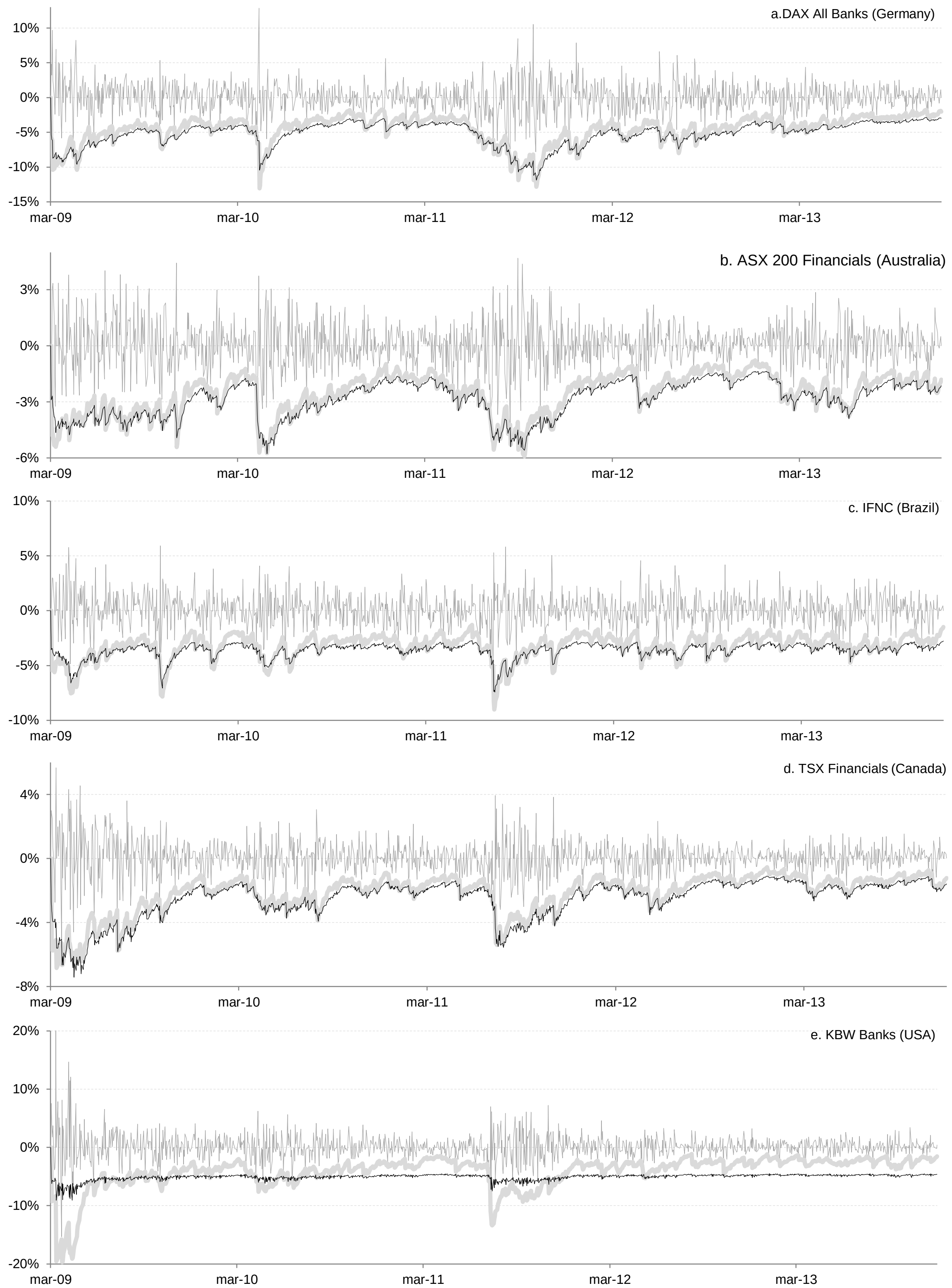


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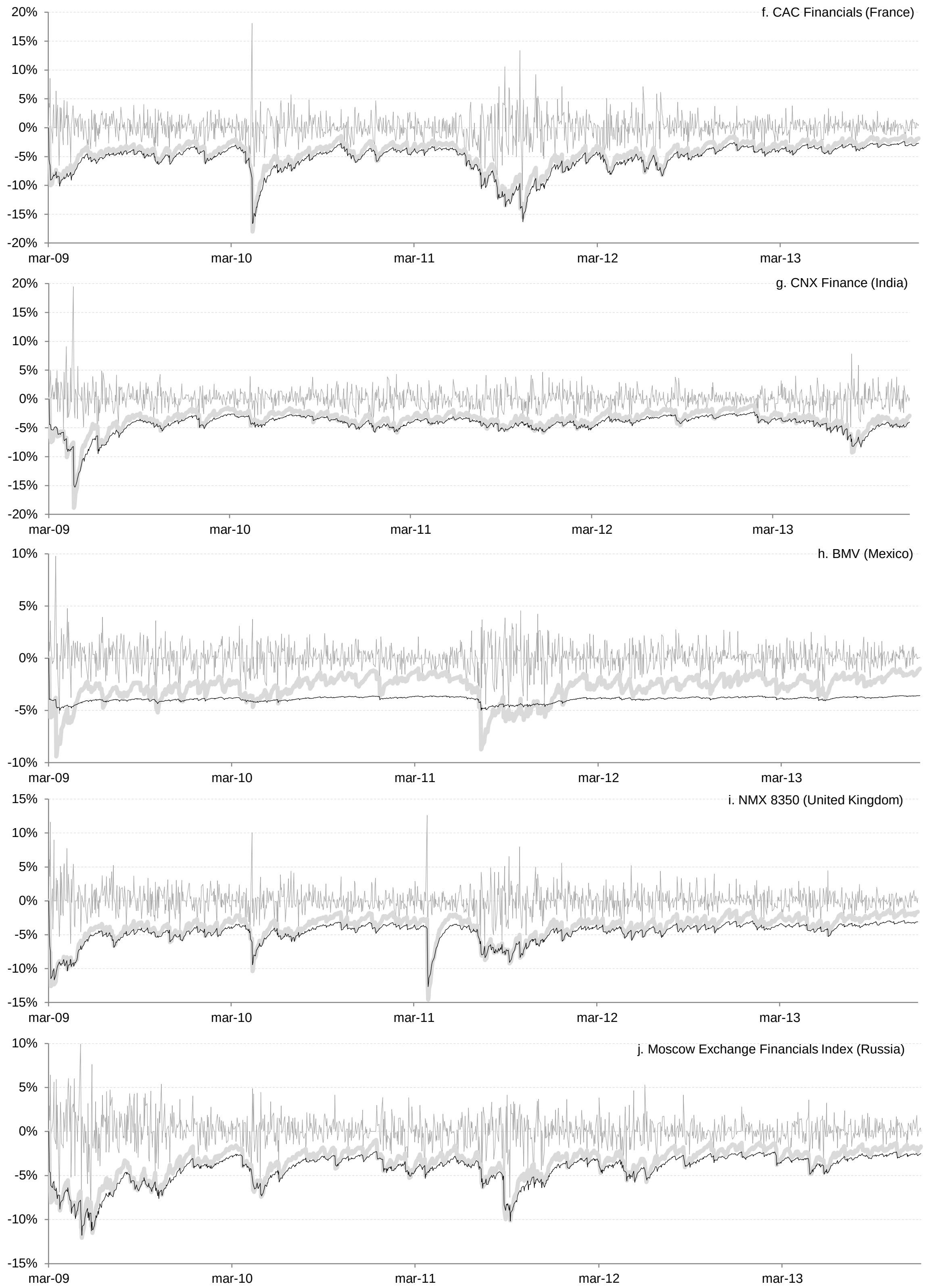
Figura 3 - VaR of returns on main worldwide financial sector indices [a]



[a]This figure plots the daily series of absolute VaR^{MBFC} (thin black line) and VaR^{Bas} (thick gray line), both with 99% of confidence level one day ahead, during the period from March 30, 2009 to December 31, 2013.

Table 4 - Backtesting methods applied to VaR of returns on main worldwide financial sector indices [a,b,c,d]

		Basel test (1255 daily observations)			Partial statistics		Lopez test	Kupiec test [c] (crit. Value X^2 (1) = 6.63)		Christoffersen test [d] (crit. value X^2 (1) = 6.63)		Kupiec-Christoffersen test [d] (crit. Value X^2 (1) = 9.21)	
Index (Country)	VaR specification	Number of violations	%	Result	Excess conservatism	Avreage violation	Result	Statistic	Result	Statistic	Result	Statistic	Result
DAX All Banks (Germany)	Basel	28	2.231%	reject	5.472%	0.113%	0.161%	14.26	reject	- x -	not applic	- x -	not applic
	Univariate	7	0.558%	no reject	5.855%	0.047%	0.028%	2.94	no reject	- x -	not applic	- x -	not applic
	Multivariate	5	0.398%	no reject	5.906%	0.046%	0.026%	5.93	no reject	- x -	not applic	- x -	not applic
ASX 200 Financials (Australia)	Basel	25	1.992%	reject	3.083%	0.090%	0.099%	9.70	reject	0.41	no reject	10.11	reject
	Univariate	25	1.992%	reject	3.230%	0.073%	0.064%	9.70	reject	0.41	no reject	10.11	reject
	Multivariate	14	1.116%	reject	3.283%	0.054%	0.037%	0.17	no reject	2.13	no reject	2.30	no reject
IFNC (Brazil)	Basel	24	1.912%	reject	3.797%	0.130%	0.213%	8.34	reject	- x -	not applic	- x -	not applic
	Univariate	8	0.637%	no reject	4.014%	0.104%	0.136%	1.90	no reject	- x -	not applic	- x -	not applic
	Multivariate	7	0.558%	no reject	4.022%	0.110%	0.153%	1.90	no reject	- x -	not applic	- x -	not applic
TSX Financials (Canada)	Basel	25	1.992%	reject	2.772%	0.096%	0.116%	9.70	reject	- x -	no reject	- x -	no reject
	Univariate	19	1.514%	reject	2.978%	0.098%	0.12%	2.90	no reject	- x -	no reject	- x -	no reject
	Multivariate	10	0.797%	no reject	3.041%	0.064%	0.052%	0.56	no reject	- x -	no reject	- x -	no reject
KBWBank (USA)	Basel	25	1.992%	reject	5.593%	0.196%	0.481%	7.07	reject	- x -	not applic	- x -	not applic
	Univariate	13	1.036%	reject	5.539%	0.296%	0.478%	0.02	no reject	- x -	not applic	- x -	not applic
	Multivariate	8	0.637%	no reject	6.122%	0.145%	0.264%	1.90	no reject	- x -	not applic	- x -	not applic

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Table 4 - Backtesting methods applied to VaR of returns on main worldwide financial sector indices [a,b]

		Basel test (1255 daily observations)			Partial statistics		Lopez test	Kupiec test [c] (crit. value $X^2(1) = 6.63$)		Christoffersen test [d] (crit. Value $X^2(1) = 6.63$)		Kupiec-Christoffersen test [d] (crit. value $X^2(1) = 9.21$)	
Index (Country)	VaR specification	Number of violations	%	Result	Excess conservatism	Avreage violation	Result	Statistic	Result	Statistic	Result	Statistic	Result
CAC Financials (France)	Basel	25	1.992%	reject	5.654%	0.117%	0.172%	9.70	reject	-x-	no reject	-x-	no reject
	Univariate	8	0.637%	no reject	6.246%	0.066%	0.055%	1.90	no reject	-x-	no reject	-x-	no reject
	Multivariate	7	0.558%	no reject	6.344%	0.051%	0.033%	2.94	no reject	-x-	no reject	-x-	no reject
CNX Finance (India)	Basel	25	1.992%	reject	4.557%	0.147%	0.271%	9.70	reject	-x-	not applic	-x-	not applic
	Univariate	10	0.797%	no reject	5.047%	0.099%	0.122%	0.56	no reject	-x-	not applic	-x-	not applic
	Multivariate	8	0.637%	no reject	5.071%	0.077%	0.078%	1.90	no reject	-x-	not applic	-x-	not applic
BMV (Mexico)	Basel	25	1.992%	reject	3.394%	0.159%	0.316%	9.70	reject	-x-	not applic	-x-	not applic
	Univariate	19	1.514%	reject	3.713%	0.161%	0.302%	2.90	no reject	-x-	not applic	-x-	not applic
	Multivariate	12	0.956%	no reject	3.751%	0.137%	0.237%	0.02	no reject	-x-	not applic	-x-	not applic
NMX 8350 (United Kingdom)	Basel	24	1.912%	reject	4.770%	0.090%	0.101%	8.34	reject	-x-	not applic	-x-	not applic
	Univariate	7	0.558%	no reject	5.244%	0.045%	0.025%	2.94	no reject	-x-	not applic	-x-	not applic
	Multivariate	8	0.637%	no reject	5.340%	0.045%	0.025%	1.90	no reject	-x-	not applic	-x-	not applic
Moscow Exchange Financials Index (Russia)	Basel	24	1.912%	reject	4.509%	0.170%	0.363%	8.34	reject	0.50	no reject	8.84	no reject
	Univariate	12	0.956%	no reject	4.948%	0.127%	0.203%	0.02	no reject	-x-	not applic	-x-	not applic
	Multivariate	10	0.797%	no reject	5.002%	0.115%	0.169%	0.56	no reject	-x-	not applic	-x-	not applic

[a] We perform the backtestings reported here to daily series of absolute VaR^{Bas}, VaR^{UBFC} and VaR^{MBFC}, with 99% of confidence level one day ahead, during the period from March 30, 2009 to December 31, 2013. We may reject the VaR specification whether the test statistic is higher than the critical value. [b] Unconditional coverage test proposed by Kupiec (1995), with confidence around 99% region defined by a ratio of log-likelihood having chi-square asymptotic distribution with one degree of freedom under the null hypothesis that the level of VaR confidence is the real likelihood of losses. [c] Unconditional coverage test proposed by Christoffersen (1998), with the region of confidence of approximately 99%, defined by a ratio of log-likelihood which has asymptotic chi-square with one degree of freedom under the null hypothesis that the exceptions are independent serially. [d] Test set of conditional and unconditional coverage, with confidence of approximately 99% region, defined by a ratio of log-likelihood having chi-square asymptotic distribution with two degrees of freedom under the null hypothesis that the confidence level of the VaR it is the real likelihood of losses and the exceptions are serially independent.

4. CONCLUSION

The active behavior against risk management by the financial sector, international committees and policy makers need to be aligned with the theoretical and empirical literature, since different research routes suggest refined statistical frameworks. In this context, as limited as noting that CAC Financials index of France has the largest standard deviation, while German DAX All Banks has the highest drawdown during the period from March 30, 2009 to December 31, 2013, seems to be the use of VaR measures based on strongly rejected assumptions about homoscedasticity and normality.

Our innovative metric of risk management not only relaxes the unreliable assumptions but also takes into account the effects of contagion and integration.

According to our findings for most relevant financial sector indices, we believe to have offered theoretical and empirical evidences that to ignore cross-effects may be unsuitable for some specific samples of assets due to effects of interdependence between financial markets. Not even the exponential character added to the VaR used in Basel seems to be sufficient to make this risk management metric able to predict crises, regulate the market or direct bank treasuries provision policies. We hope that our theoretical innovation will be useful to this literature by motivating researchers that intend to extend the traditional VaR measure.

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We add to the risk management debate concerning extensions of Value-at-Risk (VaR), following a research route that relaxes the unreliable statistical assumptions. We propose an innovative VaR measure based on time-varying moments of a best fitting distribution extracted using multivariate ARMA-GARCH. We provide a VaR that is able to capture the cross-effects associated with contagion and integration. This refined statistical risk metric is useful for samples of assets where the influence of common risk drivers should not be negligible. We implement an empirical exercise applying Basel VaR and our VaR, with and without the cross-effects in a sample of the main worldwide financial sector indices of G20 economies, covering a period sufficiently extensive. According to Basel backtesting, we reject Basel VaR in all economies and univariate VaR in four cases. The multivariate VaR is rejected in only one case: ASX 200 Financials in Australia. According to backtesting that deals with the frequency and conditionality of losses exceeding VaR, while Basel VaR is rejected for all ten economies, we fail to reject our multivariate VaR for all assets. Except for IFNC index in Brazil, our multivariate VaR shows the best performance according to the average violation and Lopez (1999) ranking criteria.

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