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# The role of household debt and delinquency decisions in consumption-based asset pricing

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**ABSTRACT**

I add to asset pricing theory by incorporating household debt and delinquency decisions into a standard quantitative model of lifecycle consumption-saving-investment. Deriving this extended system of Euler equations does not alter consumption-based fundamental asset pricing equation. It imposes that the pricing kernel has also to account jointly for two new first-order conditions. I perform empirical exercises aiming to account for equity premium in U.S. and to price six Fama-French portfolios. I find significant elasticity ranging from 0.15 to 0.75 and risk aversion from 1.33 to 3.51. These investor decisions may be playing a relevant role in terms of completing markets.

KEYWORDS: Household underlying principles, Complete Markets, Asset pricing puzzles

JEL CODES: C36, D14, G12.

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IN THIS PAPER, I analyze theoretical and empirical asset pricing implications of the consumption capital asset pricing model (CCAPM) incorporating investor decisions on credit and default.

This general equilibrium approach was originally developed and explored empirically by Samuelson (1969), Rubinstein (1974), Hall (1978), Lucas (1978), Breeden (1979) and Grossman and Shiller (1981). This framework widely used in macroeconomic theory was the natural candidate to price an asset in an equilibrium setting with risk averse agents. Over the last thirty years, it has become a standard workhorse for consumption-based analysis of financial markets stylized facts. In short, CCAPM is one the most important advances in modern finance theory.

The most famous domestic financial markets anomaly is characterized by two empirical phenomena: the large equity premium and the low risk-free rate in U.S. More specifically, the Equity Premium Puzzle (EPP) is how Mehra and Prescott (1985) labelled the systematic failure of the standard power-utility CCAPM to account for the stock market risk premium in U.S., for reasonable preference parameters. Somewhat disappointingly, this is a robust puzzle. Many studies using mostly nonlinear generalized method of moments (GMM) have found insignificant incompatible high values for the relative risk aversion and low values for the elasticity of intertemporal substitution in consumption of the representative agent. Hall (1988) claims that this parameter is unlikely to be above 0.1, and may be zero. This statement predominantly influences the discussion about this elasticity until today, according to Thimme's (2017) survey.

These robust findings suggest the traditional canonical consumption model's inability to account for its overidentifying restrictions since these are central parameters in models of dynamic choice in macroeconomics and finance. Moreover, previous research has cast doubt on whether a single asset pricing model was capable of correctly pricing the equity premium or to account for other domestic market puzzles or even puzzles in international macroeconomics. This perception led to the emergence of some empirical and theoretical strategies with respect to: i) estimation technique based on linear or nonlinear instrumental (or not) variable regressions; ii) instrument set dealing with endogeneity and the predictability of macroeconomic and financial variables; iii) the data used, i.e. sample of assets, time period and frequency or geographical origin of the data and iv) assumptions on frictions as liquidity constraints or stock market participation.

However, this literature suggests the studies in macro-finance as more promising to explain such stylized facts. According to Cochrane's (2017) survey, one of the most interesting research routes explores the consumer's utility function. Some prominent examples are catching up with the Joneses theory of Abel (1990), the recursive utility developed by Epstein and Zin (1991) and the habit formation suggested by Campbell and Cochrane (1999). Regarding the asset pricing implications of dealing with types of risk, one can see the effects of long-run risks proposed by Bansal and Yaron (2004), and idiosyncratic risks in Constantinides and Duffie (1996) and Krueger and Lustig (2010).

Some studies have analyzed only consumption data. Yogo (2006) addresses the relevance of disaggregating consumption among services, non-durable and durable goods, while Ferreira and Moore (2015) analyze the role of the variance of foreign and domestic consumptions, for instance. More recently, Kroencke (2017) analyzes the properties of consumption data without measurement error and filtering and quantifies implications for asset pricing puzzles. [1]

In this context, Brandt et al. (2006) claim that what we lack is a proper consumption-based model yet to be written, which, if found, could solve these asset pricing puzzles. I add to this debate by proposing a fresh look into the most primitive choices of the representative agent. I allow the household to decide not only how to save, consume and invest. He also chooses how to borrow and if he will default on this obligation. As a consequence, I derive an extended system of Euler equations which incorporates two new first-order conditions. It does not alter fundamental asset pricing equation, does not involve higher complexity and exempts me from making additional assumptions.

I need to make it very clear that I do not claim originality in addressing credit and default decisions in a general equilibrium framework. This approach has been used for normative and positive analysis in Chatterjee et al. (2007),

[1] There are also relevant studies in the asset pricing literature accounting for rare disasters, effects of accounting variables or behavioral finance.

Gerlach-Kristen and Merola (2018) and Athreya et al. (2018), for instance. However, this paper is the first, to my knowledge, to address the relationship between decisions on credit and default and asset pricing in a consumption-based framework. This theoretical innovation does not rival or compete with the extensive literature mentioned above. On the contrary, it complements and can be associated with any of these contributions.

Theoretically, this approach enables me to discuss on the role of issuing an unsecured debt in $t$ and the possibility of a delinquent behavior in $t+1$ given a known punishment in terms of endowment garnishment. Realize that the investor can choose the payoff after the state of nature was revealed since he is allowed to decide whether he want to repay his debt as promised or to skip debt payments without seeking formal bankrupt protection. This last option is called delinquency, a kind of partial or even total default. I claim that it works as an additional and versatile security able to span contingent claims and complete markets. This is not an Arrow security whose payoffs are contingent on the realization of the aggregate shock, but a security with endogenous payoff.

In practice, I aim to figure out the value of assets through estimating a nonlinear extended system of Euler equations characterized by consumption-saving-portfolio-loan-delinquency decisions. I implement an empirical exercise to account for equity premium in U.S. from 1987:1 to 2018:1. I avoid endogeneity critique discussed in Hall (1988). I also deal with da Costa et al.'s (2016) argument on the contrasting performance of the asset pricing tests in level and in its log-normalized versions.

To summarize the main findings, estimating the extended system of Euler equations enables me to find individual and jointly significant parameters of usual preferences given different instrument sets. I am able to find elasticity between 0.24 and 0.55 and risk aversion lower than 2.3. There seems to be no sign orthogonality misspecification. However, when the model omits household decisions on loan and default, most individual parameters are statistically zero, at the 5% significance level.

As a robustness check, I account for other domestic market stylized facts that usually escape consumption-based approaches. I price correctly returns on the six Fama and French (1993) dynamic portfolios formed on size and book-to-market. Once more, I am able to find reasonable and significant values for the main parameters. I believe I have offered evidence able to motivate the exploration of the role played by decisions on credit and default as ways to circumvent empirical difficulties of consumption-based model. Moreover, this framework enables me to discuss on credit cycles and to propose a new rule of thumb relating preferences parameters and credit variables.

This paper is structured as follows. In the Section I there is a review the literature, while Section II illustrates the setup of the model. Section III analyses the dataset and reports main findings. Section IV discuss implications for the credit market and Section V is devoted to final remarks.

## I. Literature on General Equilibrium with Credit and Default

Addressing credit decisions and its restrictions based on a neoclassical growth model with uninsurable shocks in a general equilibrium framework is not a novelty. One can see some of the early theoretical contributions in Imrohoroğlu (1989) and Aiyagari (1994), for instance.

Aiming to add to the related positive discussion, Chatterjee et al. (2007) analyze a general equilibrium model with unsecured consumer credit that incorporates the main characteristics of U.S. consumer bankruptcy law. Their calibration exercise is useful to characterize default behavior in terms of earnings for a given set of household characteristics. Using Dynamic Stochastic General Equilibrium (DSGE) model, Gerlach-Kristen and Merola (2018) find that when indebted households start facing credit constraints, due to falling property prices, they stop smoothing consumption and start deleveraging. In terms of normative properties, Dávila et al. (2012) suggest a characterization of constrained efficiency based on the first-order condition of a constrained planner's problem. It enables them to illustrate the dynamics of accumulation of capital in steady state in U.S.

Concerning more recent discussion specifically on delinquency and bankruptcy, Athreya et al. (2015) propose measuring the relative roles of recent bankruptcy reform and labor market risk in accounting for consumer debt and default over the Great Recession. Mitman's (2016) model explains the observed cross-state variation in consumer delinquency rates and it is also useful to measure the impact on bankrupt rates of two major debt-relief policies in U.S. In Athreya et al. (2018), the authors develop a quantitative model of debt delinquency and bankruptcy aiming to accommodate the dynamics informal consumer debt default.

To summarize, this theoretical and empirical agenda on credit and default decisions based on general equilibrium seems to be consistent and promising. However, probably due to the more contained evolution in credit before the 80's or the lack of available data on default on loans (from 1987) and on credit card (from 2001), for instance, this research agenda of decisions on credit and delinquency has not been prioritized by asset pricing researchers.

Only recently, the severity of the last financial crisis and the following deep recession has revived interest in the links between asset prices and credit market.

In Bordo and Jeanne (2002), the authors suggest an interesting pass-through, according to which higher credit availability boosts asset prices through liquidity and the expectation of further rises in these prices motivates raising debt. However, during periods of falling asset prices – useful as collateral – one can expect expenditure cut back and borrowing reduction.

Another relevant contribution in this context is Chen et al. (2012). They propose using a multivariate analysis accounting for the phase shift mechanism, which enables them to identify causality between financial cycles and business cycles even with raw data at different frequencies. According to their main findings, at the business cycle frequency for U.S. over the period from 1965:1 to 2010:3, real output and real stock prices tend to lead effective federal funds rate and total credit in a pro-cyclical fashion.

Even more aligned to my purpose is Berndt (2015). This author uses reduced-form models of default calibrated to expected default losses and co-movements between default losses and an equity-based pricing kernel considering market frictions. He aims to account for historical CDS spreads, while I address here the specific relationship between non-state-contingent debt, state-contingent default and consumption-based asset pricing.

For my purpose to incorporate household decisions on how to borrow and whether skipping debt payments given observable finance rate on loans, I analyze the evolution over time of these variables reported in Figure I. These time series were extracted from Federal Reserve Economic Data (FRED).

During the period from 1987:1 to 2018:1 real finance rates on personal loans range from 1.21% to 3.93%. During NBER recessions U.S. (highlighted in the shaded areas) the volatility of these rates ranges from 030% to 1.03%. However, during periods without recession its standard deviation ranges from 0.19% to 0.31%. With respect to delinquency rates, its average throughout the period is 3.25%. What is striking is the growing trend in crises. These rates have increased during such recessions, especially in the most recent one – more than 1.1% in 18 months. Moreover, during this last recession delinquency rate on single-family residential mortgages have increased more than 5%, reaching a level higher than 8% right after the recession and higher than 11% in 2010.[2]

Concerning personal loan growth, I may highlight the high volatility during the last recession, 2.9% and mainly after such crises, with a standard deviation of 6.3%. Meanwhile, consumption growth volatility reaches its highest value during the first recession, 0.43%.

These summary statistics, the pass-through between credit level and asset prices suggested by Bordo and Jeanne (2002) and the causality between financial cycles and business cycles identified in Chen et al. (2012) motivate me to better understand the relationship between credit decisions and asset prices. My difference in relation to these contributions is the framework used and consequently the theoretical and empirical analysis in terms of asset pricing implications.

I introduce the household's debt and delinquency decisions into a standard quantitative model of lifecycle consumption-saving-investment. In the original setup of this kind of general equilibrium model, the investor has to

[2] This variable is not used in the empirical exercise performed here and is not reported in Figure I.

decide his current and future values of consumption and how to buy or sell of the assets available. I allow him to endogenously determine how to borrow in $t$, given an observable finance rate on loans in $t$. More specifically, an investor may issue one-period unsecured debt with a face value which is non-state-contingent. In $t+1$ investor is allowed to decide whether he want to repay his debt as promised or to skip debt payments without seeking formal bankrupt protection. This behavior is defined in this literature as delinquent.

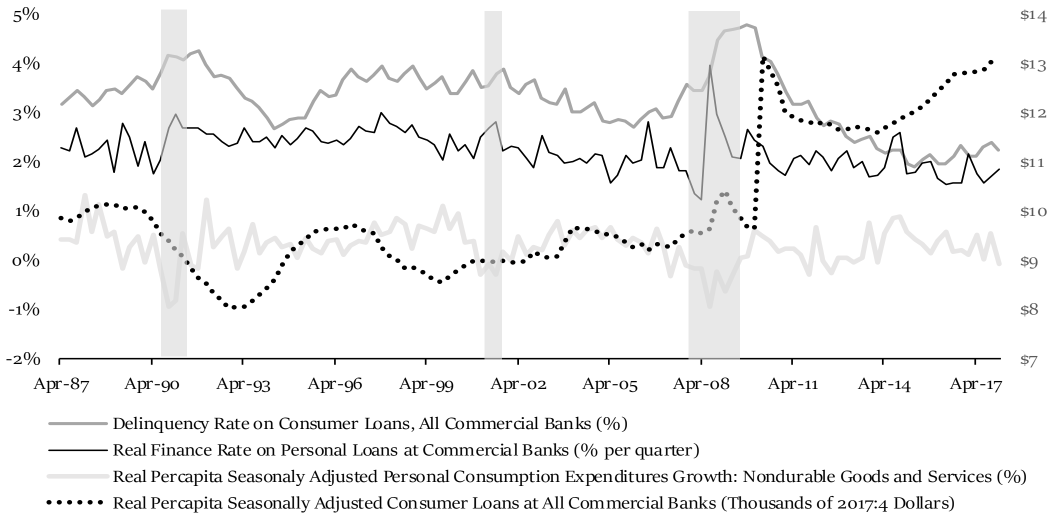


**Figure I. Household loan and consumption**. This figure presents the personal consumption growth and volume, finance rate and delinquency of household loans for U.S. over the period from 1987:1 to 2018:1. Data source: FRED.

I follow Athreya (2012) by imposing a punishment to default. The delinquent investor faces endowment garnishment: the percentage of endowment available is a linear function of the default decision. I also allow him to roll the debt under the same conditions in which he grants a new residual debt. Concerning the personal bankruptcy, I recognize the existence and the relevance of this formal procedure that removes unsecured debt obligations subject to some specific cases. However, I rely on one of the main findings reported in Athreya et al. (2009) to rule out this possibility. They find that unsecured credit markets pass-through increased income risk to consumption in U.S., irrespective of bankruptcy policy.

Following the related literature on consumption-based asset pricing, my purpose here is not characterizing the equilibrium, so I do not need to impose any no-Ponzi game condition useful to guarantee such existence. My intent is characterizing the conditions under which I am able to price correctly assets, i.e. claims on a stream of payments. I analyze such conditions by estimating the value and inferring on the individual and joint significance of the preference parameters, besides overall fitting of non-linear preference-based models.

## II. Model

The single most important advance in asset pricing from an economist's perspective was the development of the CCAPM associated with Rubinstein (1974), Lucas (1978), Hall (1978), Breeden (1979) and Grossman and Shiller (1981).

Concerning the hypothesis to derive a consumption-based pricing kernel, one has not to assume complete markets or the existence of a representative consumer. First-order conditions yet to be derived apply to each individual investor and for each asset to which he has access. However, from an empirical perspective, one may assume the existence of a representative investor, given the need to use aggregate consumption data in the utility function.

Given complete markets, aggregation is not needed for the existence of a representative agent. This is true even if individuals are heterogeneous in preferences and wealth, provided that they have the same time discount rate and beliefs. Moreover, the representative individual's relative elasticity is no larger than the most risk averse individual and no smaller than the least risk averse individual.

Still with respect to the hypothesis, I follow the didactic Cochrane's (2001) contribution. He argues that I do not have to assume anything on returns distribution and this pricing equation must hold for any asset, stock, bond, option, etc. This fundamental relation holds for any two periods of a multiperiod model and I do not have to assume i.i.d. returns over time. I do not assume that investors have no nonmarketable human capital or no outside sources of income.

*A. Canonical CCAPM*

Consider an economy endowed with an infinitely lived representative consumer whose preferences are representable by a von Neumann-Morgenstern utility function $u(.)$ twice continuously differentiable. The canonical problem of the representative agent is given by intertemporal and cross-state decisions on consumption, saving and investment according to

$$Max_{\xi}\, u(C_t) + \mathbb{E}_t[\textstyle\sum_{j=1}^{+\infty} e^{-\delta j} u(\boldsymbol{C}_{t+j})] \text{ s. t.} \tag{1}$$

$$C_t = e_t - \boldsymbol{p}_t \boldsymbol{\xi}_t \tag{2}$$

$$\boldsymbol{C}_{t+j} = \boldsymbol{e}_{t+j} + \boldsymbol{X}_{t+j} \boldsymbol{\xi}_{t+j-1} - \boldsymbol{p}_{t+j} \boldsymbol{\xi}_{t+j} \tag{3}$$

This is a convenient mathematical formalism to model what an investor wants. Let me formalize it in the following way. $\mathbb{E}_t(.)$ denotes the conditional expectation given the available information in $t$. The timing is such that in $t$, household decides current consumption, $C_t$, given his current endowment, $e_t$ and current vector of $N$ asset prices available, $\boldsymbol{p_t}$**.** Future consumption, $\boldsymbol{C}_{t+j}$, is random. The agent does not know his future endowment which is contingent on $S$ possible states of nature, as well as $S$x$N$ matrix of assets payoffs, given by $\boldsymbol{X}_{t+j}$.

The utility function captures the fundamental desire for more consumption, rather than posit a desire for other purposes. More specifically, the objective function (1) captures investors' impatience, by discounting the future by $e^{-\delta}$. I can quantitatively correct for delay of cash flows by estimating $\delta$, the consumer's subjective time discount rate. Also, I need to worry about the parameters associated with each preference specification.

Now, assume that investor can freely trade as much of the assets as he wishes given the unique prices. Denote by $\xi_t^i$ the amount of the asset $i$ investor chooses to buy in $t$. Substituting the constraints into the objective function and setting the derivative with respect to portfolio equal to zero, I obtain the first-order condition for an optimal consumption-saving-portfolio choice,

$$1 = \mathbb{E}_t\left[e^{-\delta}\frac{u'(C_{t+1})}{u'(C_t)}\boldsymbol{R}_{t+1}^i\right], \forall i = 1, \dots, N \tag{4}$$

where $\boldsymbol{R}_{t+1}^i$ is the real gross return on asset $i$ in $t+1$ given by $\boldsymbol{x}_{t+1}^i/p_t^i$. This is the well-known consumption-based version of fundamental asset pricing equation. In this micro fundamented description of the world, the stochastic discount factor (SDF), $\boldsymbol{M}_{t+1}$, is given by the discounted ratio of consumption marginal utilities.

Although this consumption-based approach is very interesting from an economist's perspective, macro-finance literature is exploring different utility functions, revisiting consumption data or modelling the endowment risks as ways to deal with the empirical difficulties of canonical CCAPM.

However, in the quest towards the resolution of the EPP and other puzzles in finance, the household decisions on borrowing and delinquency have not been introduced into standard general equilibrium model of lifecycle consumption-saving-investment. I address this specific issue by providing theoretical and empirical asset pricing implications of deriving an extended system of Euler equations which incorporates two new first-order conditions. I now discuss each of the additional assumptions credit decisions in detail to highlight and explain the timing of the model.

*B. Revisiting CCAPM: Model of Lifecycle Consumption-Saving-Investment-Debt-Delinquency*

First, following the canonical model, it is required that investor still lives infinitive periods, receives stochastic endowments and has free portfolio formation.

Second, I assume that an investor wishing to borrow in $t$ may issue one-period unsecured debt with a non-state-contingent face value given by $b_t$. Household issues all unsecured debt to a single lender given a known finance rate $r_t$. This assumption is identical to the one usually assumed in most of the studies which address credit in a general equilibrium framework.

Third, I need to introduce delinquent behavior into this quantitative model of lifecycle consumption-saving-investment-debt. In the following period, $t+1$, the capitalized debt is $b_t.(1+r_t)$. Following Athreya (2012), for instance, the investor has the possibility of default, i.e. paying $b_t.(1+r_t).(1-\boldsymbol{d}_{t+1})$. In other words, the household has two options: repaying debts as promised, $\boldsymbol{d}_{t+1}=0$, or he can simply not repay the debt as promised, $0<\boldsymbol{d}_{t+1}\leq 1$, without seeking formal bankrupt protection. This second option is called delinquency, a kind of partial or even total default, given by $\boldsymbol{d}_{t+1}=1$.

Fourth, I also follow the related literature by assuming that the financial system imposes a punishment against default behavior. The delinquent household faces endowment garnishment in $t+1$ given by $\boldsymbol{e}_{t+1}.\lambda.\boldsymbol{d}_{t+1}$. More specifically, the percentage of endowment available is a linear function of the default decision given by $(1-\lambda.\boldsymbol{d}_{t+1})$.

Finally, after this punishment measured by the parameter $\lambda$, but still in the same period, in $t+1$, he can roll the debt $b_t.(1+r_t).\boldsymbol{d}_{t+1}$ under the same conditions in which he grants a new residual debt, given by $\boldsymbol{b}'_{t+1}$. In other words, in $t+1$ household decides a total debt level, given by

$$\boldsymbol{b}_{t+1}=b_t.(1+r_t).\boldsymbol{d}_{t+1}+\boldsymbol{b}'_{t+1} \tag{5}$$

In this more realistic context, I must replace original budget restrictions (2) and (3) for

$$C_t=e_t-\boldsymbol{p}_t\boldsymbol{\xi}_t+b_t \tag{6}$$

$$\boldsymbol{C}_{t+j}=\boldsymbol{e}_{t+j}(1-\lambda.\boldsymbol{d}_{t+1})+\boldsymbol{X}_{t+j}\boldsymbol{\xi}_{t+j-1}-b_t.(1+r_t).(1-\boldsymbol{d}_{t+1})+\boldsymbol{b}'_{t+1}-\boldsymbol{p}_{t+j}\boldsymbol{\xi}_{t+j} \tag{7}$$

Substituting (5) in (7), after rolling the debt, one can rewrite future budget constraint as

$$\boldsymbol{C}_{t+j}=\boldsymbol{e}_{t+j}(1-\lambda.\boldsymbol{d}_{t+1})+\boldsymbol{X}_{t+j}\boldsymbol{\xi}_{t+j-1}-b_t.(1+r_t)+\boldsymbol{b}_{t+1}-\boldsymbol{p}_{t+j} \tag{8}$$

Once more, substituting the constraints into the objective function and setting the derivative with respect to $b_t$ and $\boldsymbol{d}_{t+1}$ equal to zero, I obtain the following respective first-order conditions for an optimal consumption-saving-portfolio-debt-delinquency choice,

$$1=\mathbb{E}_t\left[e^{-\delta}\frac{u'(C_{t+1})}{u'(C_t)}(1+r_t).(1-\boldsymbol{d}_{t+1})\right] \tag{9}$$

$$0=\mathbb{E}_t[e^{-\delta}u'(\boldsymbol{C}_{t+1})(b_t.(1+r_t)-\lambda.\boldsymbol{e}_{t+j})] \tag{10}$$

Based on this extended set of household choices, I propose estimating the system of Euler equations given by fundamental asset pricing equation (4) for each one among $N$ assets available in the economy, which remains unchanged. In addition, the consumption-based pricing kernel must also account for first- order conditions (9) and (10).

First-order condition (9) applies to each individual investor and for the unique debt option to which he has access. Comparing it with condition (4), here I need an adjusted (by non-delinquency) stochastic discount factor to price real gross rate on the consumer loan. Finally, I must highlight the contingent nature of such decisions. Issuing in $t$ one-period unsecured debt is non-state-contingent while in $t+1$ he faces the possibility of a state-contingent default. In this sense, I claim that given this property it works as a versatile asset, which can be useful to span or synthetize contingent claims and thus completing markets.

## III. Revisiting Domestic Market Stylized Facts

Kocherlakota (1996) assess theoretical and empirical attempts to address stock market premium in U.S. The author argues that while there are some plausible explanations for the low level of short-term Treasury Bill returns, the large equity premium is still a puzzle to finance.

In this context, there are two relevant tests that can be performed in a GMM setup. Following the related literature, first I use the canonical procedure by testing the conditional moment restrictions associated with (4) aiming to price jointly real gross return on S&P 500 and 90-day Treasury Bill in U.S. For my purposes to evaluate the extended system of first-order conditions in the second test, I just add moment restrictions (9) and (10). This pricing test procedure enables me to infer about the absolute performance of the extended system of moment conditions. It also allows us to compare the main results with those found using the canonical system.

The results of both testes are reported in Tables II and III, respectively.

Reassured that the extended system of Euler equations does a good job in accounting for equity premium, for reasonable preference parameters and overall fitting, I proceed to a second exercise. To account for the cross-section behavior of domestic assets, I also perform pricing tests for the six Fama and French (1993) benchmark dynamic portfolios extracted from the Fama–French library. These results are reported in Table IV. For both exercises, I use two preferences. Their specifications are described in the following subsection.

*A. Preferences Specification*

Under a consumption-based approach, it is usual to estimate the utility function parameters and then to test the associated system of orthogonality restrictions using GMM. I use as a benchmark the standard model of consumer preferences (CRRA), which is explored empirically by the majority of articles about elasticity in consumption. This preference it is scale-invariant and if agents have different levels of wealth but the same utility, then this will also be the utility of the representative agent. Although it has been extensively used in finance due to its empirical, analytical and intuitive convenience, it really does not work well in practice, there being evidence of its incapability to account for stylized facts. The SDF of this utility ($M_{t+1}^{CRRA}$) is given by

$$M_{t+1}^{CRRA} = e^{-\delta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}, \text{if } \psi \neq 1 \text{ and } M_{t+1}^{CRRA} = e^{-\delta}\frac{C_t}{C_{t+1}}, \text{if } \psi = 1 \tag{11}$$

The main disadvantage of (11) is that the Arrow-Pratt measure of relative risk aversion, $\gamma$, is the inverse of the agents' elasticity of intertemporal substitution in consumption parameter, $\psi$. Thus, allowing for high risk aversion to account for the equity premium implies to accept a very low intertemporal marginal rate of substitution. As a consequence, whenever one is willing to accept the high coefficient of relative risk aversion that is needed to correctly price the equity premium, the risk-free puzzle arises.

To avoid this trap Epstein and Zin (1991) define more general preferences which preserve many of the attractive features of power utility as the scale-invariance and disentangle both parameters. The SDF of this utility ($M_{t+1}^{EZIN}$) is given by

$$M_{t+1}^{EZIN} = e^{-\delta\theta} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} \boldsymbol{R}_{w,t+1}^{\theta-1} \tag{12}$$

where $\theta = \frac{1-\gamma}{1-\psi^{-1}}$ and $\boldsymbol{R}_{w,t+1}$ denotes the return on the wealth portfolio.

*B. Data and summary statistics*

In terms of sample size, the main limitation for the time-series span used here regards the credit variables; delinquency rate on consumer loans are available only since 1987, for instance.

The largest sample covers the period from 1978:1 to 2018:1, at a quarterly frequency, comprising 125 observations. Macroeconomic and financial variables were extracted from Federal Reserve Economic Data (FRED). Except for the delinquency rate all nominal variables were transformed into real using the corresponding U.S. Consumer Price Index.

Table I contains some summary statistics for all exogenous and endogenous variables.

The average risk premium on S&P 500 is 5.2% at an annual rate, a little lower than 6% observed in annual U.S. data from 1889–1978 originally studied by Mehra and Prescott (1985) and Kocherlakota (1996). As usual, the correlation of per capita consumption growth with stock returns (0.11) is only slightly bigger than its correlation with bond returns (-0.05). However, the correlation of per capita consumption growth with the real gross rate on personal loan discounted by delinquency is higher, 0.35.

Corroborating Bordo and Jeanne's (2002) pass-through I find a positive correlation between loan and real return on the stock market proxy, 0.07. Still according to previous statistics reported in this literature, all six Fama/French dynamic portfolios formed on size and book-to-market show higher values of mean and volatility than S&P500.

I emphasize the order of magnitude of the volatility of loan and delinquency endogenous variables, a crucial element in the extended household decisions framework, characterized by first-order conditions (9) and (10).

*C. GMM Estimation Setup and Instruments*

In choosing how to weight the moments used for estimation, I face a trade-off between attaining full efficiency and correctly specifying the structure of the variance–covariance matrix. To implement a test to account for the EPP based on the canonical system of restrictions given by (4) and on the extended system given by (4), (9) and (10), I will use optimal GMM. I have a reasonable number of moment conditions vis-à-vis my sample size. My estimates are produced by a procedure which performs a single weight iteration after the initial 2SLS estimates and then iterates coefficients to convergence. I use HAC (Newey-West) as the weighting matrix which is a heteroskedasticity and autocorrelation consistent estimator of the long-run covariance matrix.

As instruments, I use specific financial variables carefully chosen according to their forecasting potential. For the EPP, I follow Hansen and Singleton (1982) by using consumption growth and other endogenous and/or exogenous variables associated with the respective preference lagged one period. Considering the fact that expected returns and business cycles are correlated, I use stronger instruments as a robustness check. I use macroeconomic variables with documented forecasting ability: Dividend-Price ratio ($DP$) and Investment to Capital Stock ratio ($IC$), following Cochrane's (1996) procedure. Aiming to price Fama-French portfolios, I use endogenous and/or exogenous variables associated with the respective preference lagged one period.

**Table I. Summary statistics**

Quarterly series from 1987:1 to 2018:1 (125 observations). Data source: FRED.

| | Mean | St. dev. |
|---|---|---|
| Exogenous variables - Assets | | |
| Real Return on 3-Month Treasury Constant Maturity (% per quarter) | 0.03% | 0.52% |
| Real Return on S&P 500 (% per quarter) | 1.29% | 6.72% |
| Real Return on Fama/French portfolio #1 (% per quarter) | 2.53% | 12.39% |
| Real Return on Fama/French portfolio #2 (% per quarter) | 3.57% | 9.72% |
| Real Return on Fama/French portfolio #3 (% per quarter) | 3.80% | 10.65% |
| Real Return on Fama/French portfolio #4 (% per quarter) | 3.08% | 8.35% |
| Real Return on Fama/French portfolio #5 (% per quarter) | 2.82% | 7.82% |
| Real Return on Fama/French portfolio #6 (% per quarter) | 3.03% | 9.37% |
| Other exogenous variables | | |
| Real Return on Stock market capitalization Wilshire - wealth proxy (% per quarter) | 1.79% | 7.89% |
| Real Finance Rate on Personal Loans at Commercial Banks (% per quarter) | 2.22% | 0.40% |
| Real Percapita Seasonally Adjusted Personal Income (Thousands of 2017:4 Dollars) | $ 43.33 | $ 4.96 |
| Investment to Capital Stock Ratio (%) | 6.11% | 0.58% |
| Dividend to Price Ratio (%) | 2.19% | 0.67% |
| Endogenous variables | | |
| Delinquency Rate on Consumer Loans, All Commercial Banks (%) | 3.25% | 0.65% |
| Real Percapita Seasonally Adjusted Consumer Loans at All Commercial Banks (Thousands of 2017:4 Dollars) | $10.02 | $1.38 |
| Real Percapita Seasonally Adjusted Personal Consumption Expenditures: Nondurable Goods and Services (Thousands of 2017:4 Dollars) | $31.87 | $3.23 |

*D. Equity Premium Puzzle: Pricing Test Results*

For my purposes, there are two relevant tests that can be performed in a GMM setup.

First, I analyze whether the parameters are statistically zero based on a robust Wald test on the individual and joint significance of them in these systems. The second is a standard overidentifying restrictions procedure. J test has the usual interpretation of a test of orthogonality between the errors in each moment restriction and the instruments used in GMM estimation. In other words, it is a specification test for the validity of instruments.

Table II displays the results of traditional consumption-based kernels in pricing returns related to EPP. This model omits household decisions on loan and delinquency. I find that individual parameters $\delta$ and $\psi$ are statistically zero, at the 5% significance level, for both preferences specifications based on all of instrument sets used. Only at the 10% significance level, I am able to find some non-zero values for $\psi$ ranging from 0.21 to 0.71. The estimation of $\theta$ in Epstein and Zin (1991) preferences are significant at 1% significance, except for the first instrument set. Also, Wald-test results show almost no sign of joint significance for CRRA model, while all parameters seem to be jointly statistically non-zero at 1% level in Epstein and Zin preferences.

**Table II. Equity Premium Puzzle: Estimating Euler system of equations Model of Lifecycle Consumption-Saving-Investment**

Hansen's (1982) efficient GMM (update weights once and then iterate coefficients to convergence) used to test Euler equations and to estimate parameters over the period from 1987:1 to 2018:1. The moment restrictions are:

$$1 = \mathbb{E}_t\left[e^{-\delta}\frac{u'(\boldsymbol{C}_{t+1})}{u'(C_t)}\boldsymbol{R}^{SP}_{t+1}\right]$$

$$0 = \mathbb{E}_t\left[e^{-\delta}\frac{u'(\boldsymbol{C}_{t+1})}{u'(C_t)}(\boldsymbol{R}^{SP}_{t+1} - \boldsymbol{R}^{Tbill}_{t+1})\right]$$

Instrument sets 1 to 3 are comprised by variables associated with the model in question: 1) endogenous lagged one period; 2) exogenous variables lagged one period; 3) endogenous and exogenous variables lagged one period. Instrument set 4 is composed by $IC$ and $DP$ lagged on period. Respective p-values are reported in the box brackets. Data source: FRED. * p-value <0.05. ** p-value <0.01.

| | Preference - CRRA | | | | Preference - Epstein and Zin | | | |
|---|---|---|---|---|---|---|---|---|
| | Instrument sets | | | | Instrument sets | | | |
| Parameters | #1 | #2 | #3 | #4 | #1 | #2 | #3 | #4 |
| δ | 0.0004 | -0.0089 | 0.0023 | -0.0061 | 0.0187 | -0.0031 | 0.0019 | -0.0104 |
| | [0.8988] | [0.2989] | [0.3437] | [0.0922] | [0.2454] | [0.5604] | [0.5076] | [0.7114] |
| ψ | 1.1301 | 0.2147 | 0.7759 | 0.4715 | -0.1786 | 0.3707 | 0.7106 | 0.4332 |
| | [0.3559] | [0.0719] | [0.1115] | [0.0720] | [0.2419] | [0.0815] | [0.0931] | [0.7444] |
| θ | | | | | -2.1997 | 0.9744 ** | 0.9669 ** | 0.4495 ** |
| | | | | | [0.2624] | [0.0000] | [0.0000] | [0.2670] |
| Overall fit | 0.0271 | 0.0415 | 0.0582 | 0.0526 | 0.0052 | 0.0405 | 0.0530 | 0.0492 |
| (J-statistic) | [0.9865] | [0.9998] | [0.9999] | [0.9997] | [0.9423] | [0.9999] | [1.0000] | [0.9971] |
| Wald test - | 1.2627 | 54.0157 ** | 2.6350 | 83.7831 | 103.6811 ** | 661.6879 ** | 626.2023 ** | 29.7864 ** |
| H0: δ=0, | [0.5319] | [0.0000] | [0.2678] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] |
| ψ=0, θ=0 | | | | | | | | |

In Table III, I present the estimates of the parameters of the model incorporating household primitive decisions on loan and delinquency. To summarize, all parameters in both preferences frameworks using any of the instrument sets are individual and jointly significant at 1% level, except for $\delta$, non-zero only at 5% in Epstein and Zin model with stronger instruments. Regarding $J$ test, I cannot reject the null of the suitability of the model.

How relevant are these findings and what are they saying to us? In few words, the values of the significative parameters are closer to the values expected by theory and by calibration exercises.

First, as reported in the recent Thimme's (2017) survey is unlikely to find significant elasticity above 0.1. Comparing with previous evidence, Hansen and Singleton (1982) find $\psi$ close to unity. This study follows the empirical related literature by including once lagged consumption growth among instruments which, according to Hall (1988), would lead to endogeneity and consequently high $\psi$ estimates. I deal with Hall's criticize by circumventing endogeneity problem, i.e. I rely on exogenous instruments. However, while $\psi$ estimates in Hall's study are insignificant avoiding endogeneity, I strongly reject individual and jointly insignificance of $\psi$ when household also decide on loan and delinquency, given all instrument sets used here.

**Table III. Revisiting Equity Premium Puzzle: Estimating Euler system of equations Model of Lifecycle Consumption-Saving-Investment-Debt-Delinquency**

Hansen's (1982) efficient GMM (update weights once and then iterate coefficients to convergence) used to test Euler equations and to estimate parameters over the period from 1987:1 to 2018:1. The moment restrictions are:

$$1 = \mathbb{E}_t\left[e^{-\delta}\frac{u'(\boldsymbol{C}_{t+1})}{u'(C_t)}\boldsymbol{R}_{t+1}^{SP}\right]$$

$$0 = \mathbb{E}_t\left[e^{-\delta}\frac{u'(\boldsymbol{C}_{t+1})}{u'(C_t)}(\boldsymbol{R}_{t+1}^{SP} - \boldsymbol{R}_{t+1}^{Tbill})\right]$$

$$1 = \mathbb{E}_t\left[e^{-\delta}\frac{u'(\boldsymbol{C}_{t+1})}{u'(C_t)}(1 + r_t).(1 - \boldsymbol{d}_{t+1})\right]$$

$$0 = \mathbb{E}_t[e^{-\delta}u'(\boldsymbol{C}_{t+1})(b_t.(1 + r_t) - \lambda.\boldsymbol{e}_{t+j})]$$

Instrument sets 1 to 3 are comprised by variables associated with the model in question: 1) endogenous lagged one period; 2) exogenous variables lagged one period; 3) endogenous and exogenous variables lagged one period. Instrument set 4 is composed by $IC$ and $DP$ lagged on period. Respective p-values are reported in the box brackets. Data source: FRED. * p-value <0.05. ** p-value <0.01.

| | Preference - CRRA | | | | Preference - Epstein and Zin | | | |
|---|---|---|---|---|---|---|---|---|
| | Instrument sets | | | | Instrument sets | | | |
| Parameters | #1 | #2 | #3 | #4 | #1 | #2 | #3 | #4 |
| δ | -0.0051 ** | -0.0058 ** | -0.0043 ** | -0.0057 ** | -0.0162 ** | -0.0059 ** | -0.0066 ** | -0.0203 * |
| | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0396] |
| ψ | 0.5497 ** | 0.4368 ** | 0.5238 ** | 0.4993 ** | 0.4298 ** | 0.5148 ** | 0.4769 ** | 0.2383 ** |
| | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0004] |
| λ | 0.2354 ** | 0.2365 ** | 0.2371 ** | 0.2380 ** | 0.2363 ** | 0.2354 ** | 0.2370 ** | 0.2401 ** |
| | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] |
| θ | | | | | 0.8249 ** | 0.9352 ** | 0.8983 ** | 0.7854 ** |
| | | | | | [0.0000] | [0.0000] | [0.0000] | [0.0004] |
| Overall fit (J-statistic) | 0.1843 | 0.1862 | 0.2001 | 0.1520 | 0.1805 | 0.1903 | 0.1980 | 0.1530 |
| | [1.0000] | [1.0000] | [1.0000] | [1.0000] | [1.0000] | [1.0000] | [1.0000] | [1.0000] |
| Wald test - H0: δ=0, ψ=0, λ=0, θ=0 | $7.7 \times 10^3$ ** | $2.8 \times 10^4$ ** | $2.6 \times 10^5$ ** | $1.4 \times 10^4$ ** | $1.2 \times 10^4$ ** | $2.8 \times 10^4$ ** | $3.0 \times 10^4$ ** | $7.8 \times 10^3$ ** |
| | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] |

Moreover, I follow Lucas' (1990) rule of thumb which relates interest rates $r_f$ with a consumer's subjective time discount rate and consumption growth via

$$r_{f,t} = \delta + \psi^{-1}.\ln(\boldsymbol{C}_{t+1}/C_t) \tag{13}$$

Based on the average value of the range of my estimates of $\delta$ and the time series from 1987:1 to 2018:1, I should rule out an elasticity of intertemporal substitution below 0.31. My estimates of $\psi$ range from 0.43 to 0.55, except for Epstein and Zin specification with the last instrument set used.

Second, it is usual to find high and incompatible values for the coefficient of relative risk aversion even in the literature that has focused on developing models of investor behavior. I find significant values for Arrow-Pratt measure of relative risk aversion ranging from 1.82 to 2.29 when the household with CRRA preferences also decide on borrow and default. Comparing theses finding with some recently reported also using CRRA, Matos and da Costa (2016) find this risk aversion of around 5.8 covering the period from 1977:1 to 2004:4. In Kroencke (2017) this parameter assumes values higher than 15.1 for a longer period from 1928 to 2014 using filtered and nonfiltered consumption data extracted from National and Income Product Accounts (NIPA). Considering Epstein and Zin preferences, I find significative and reasonable estimates of risk aversion ranging from 1.88 to 3.51.

Third, a classical issue in this literature is forecasting risk-free rate given by the inverse of unconditional expectation of the SDF. Comparing the results obtained from the standard and the extended systems of restrictions given both preferences specifications, I find the lower level of mean square error when I use the extended system of Euler equations and the consumer has a CRRA preference. This error is 0.047%, while the errors are 0.365% and 0.443% when I use canonical system with CRRA and Epstein-Zin preferences, respectively.

Finally, I find that the parameter $\lambda$, related to the garnishment of a delinquent household endowment, ranges from 0.23 to 0.24. Among the few similar empirical contributions, in Athreya (2012) this parameter is assumed to be lower than 0.1 for credit card in U.S.

This finding means that, for delinquency rates ranging from 1.88% to 4.77% throughout the period, about 0.4% and 1.1% of endowment becomes unavailable for delinquent investors in U.S.

*E. Additional Pricing Test Results: Dynamic Portfolios*

Consumption-based models predict that assets with high average returns have high consumption covariances while assets with low average returns have low covariances. To deal with this evidence, the literature following mainly Fama and French (1993) has sorted firms according to their market equity and book-to-market ratio. These size and value portfolios are very much the standard test assets in this asset pricing literature. Thus, another way to corroborate the success of extended version of the consumption approach is looking at it results in empirical cross-sectional pricing tests.

Table IV reports the main results of pricing test of six Fama and French (1993) dynamic portfolios formed on size and book-to-market. In this robustness exercise, I reject at 1% significance level the null that parameters are individual zero for both preferences, except for Epstein and Zin model with first instrument set comprised by only endogenous variables. I reject the null of joint insignificancy for all cases. With regards to the overall fitting, I cannot reject the null of the suitability of the model.

I find that elasticity ranges from 0.15 to 0.75, while risk aversion ranges from 1.33 to 2.29. The values of the parameter related to the garnishment of a delinquent household endowment remain similar to those previously reported. These results suggest that I am able to account for cross-section dynamics of domestic assets in U.S, at least in terms of size and value. This is a difficult test even for the SDF mimicking portfolio statistically devised and used in da Costa, Issler and Matos (2015).

**Table IV. Revisiting other domestic market stylized facts: Estimating Euler system of equations Model of Lifecycle Consumption-Saving-Investment-Debt-Delinquency**

Hansen's (1982) efficient GMM (update weights once and then iterate coefficients to convergence) used to test Euler equations and to estimate parameters over the period from 1987:1 to 2018:1. The moment restrictions are:

$$1 = \mathbb{E}_t\left[e^{-\delta}\frac{u'(\boldsymbol{C}_{t+1})}{u'(C_t)}\boldsymbol{R}_{t+1}^{FFi}\right], i = 1, \dots, 6$$

$$1 = \mathbb{E}_t\left[e^{-\delta}\frac{u'(\boldsymbol{C}_{t+1})}{u'(C_t)}(1 + r_t).(1 - \boldsymbol{d}_{t+1})\right]$$

$$0 = \mathbb{E}_t[e^{-\delta}u'(\boldsymbol{C}_{t+1})(b_t.(1 + r_t) - \lambda.\boldsymbol{e}_{t+j})]$$

Instrument sets 1 to 3 are comprised by variables associated with the model in question: 1) endogenous lagged one period; 2) exogenous variables lagged one period; 3) endogenous and exogenous variables lagged one period. Respective p-values are reported in the box brackets. Data source: FRED. * p-value <0.05. ** p-value <0.01.

| | Preference - CRRA | | | Preference - Epstein and Zin | | |
|---|---|---|---|---|---|---|
| | Instrument sets | | | Instrument sets | | |
| Parameters | #1 | #2 | #3 | #1 | #2 | #3 |
| δ | 0.0183 ** | 0.0208 ** | 0.0208 ** | 0.4437 | 0.0302 ** | 0.0300 ** |
| | [0.0000] | [0.0000] | [0.0000] | [0.9119] | [0.0000] | [0.0000] |
| ψ | 0.7068 ** | 0.7501 ** | 0.6814 ** | 0.0042 | 0.1819 ** | 0.1555 ** |
| | [0.0000] | [0.0000] | [0.0000] | [0.9160] | [0.0000] | [0.0000] |
| λ | 0.2354 ** | 0.2361 ** | 0.2362 ** | 0.2356 ** | 0.2358 ** | 0.2359 ** |
| | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] |
| θ | | | | 0.0067 | 0.2515 ** | 0.2370 ** |
| | | | | [0.9159] | [0.0000] | [0.0000] |
| Overall fit (J-statistic) | 0.1914 | 0.2034 | 0.2050 | 0.1875 | 0.2040 | 0.2054 |
| | [1.0000] | [1.0000] | [1.0000] | [1.0000] | [1.0000] | [1.0000] |
| Wald test - H0: δ=0, ψ=0, λ=0, θ=0 | $1.4_x10^4$ ** | $1.5_x10^5$ ** | $1.1_x10^7$ ** | $2.0_x10^4$ ** | $1.2_x10^5$ ** | $1.6_x10^6$ ** |
| | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] | [0.0000] |

## IV. Discussion on Credit Market

The literature on Equity Premium and Risk-Free Puzzles is consensual in arguing that consumption is not volatile enough to generate the observed equity premium in the standard power-utility consumption-based model without very large risk aversion coefficients. In other words, expected returns on stock market are high because they fall when investors marginal utility is also high. Over the period from 1987 to 2018, the Sharpe ratio of market is 0.38 while annual consumption growth ranges from -3.8% to 5.3% and has a standard deviation of 0.8%. Their correlation is 0.11. In this statistical context, loan may be playing a relevant role. It has a volatility of 7.5% and its correlation with excess return on stock market is -0.19. I highlight that credit cycles occur more frequently and have higher breadth comparing with consumption cycles.

Besides these summary statistics, Bordo and Jeanne (2002) suggest the pass-through between credit level and asset prices while Chen et al. (2012) identify the causality between financial cycles and business cycles. This context is able to motivate the study of the role of decisions on credit and delinquency in asset pricing.

According to robust and significative results reported in Tables III and IV, the parameter $\lambda$, related to the garnishment of a delinquent household endowment, ranges from 0.23 to 0.24. It enables us to measure that about 0.4% and 1.1% of endowment becomes unavailable for delinquent investors in U.S. during the period from 1987 to 2018. Another interesting implication of this parameter is due to the sign of the first derivative $\partial b_t / \partial \lambda > 0$, i.e. the possibility to raise household credit grant given a higher value for the garnishment of endowment parameter, without reducing loan rates.

This extended version of households' primary decisions also enables me to draw the locus of possible combinations of delinquency (horizontal axis) and real per capita consumer loans (vertical axis). In Figure II, I plot such possible locus based on the estimates reported in Table III using Epstein and Zin preferences and the fourth instrument set comprised only by strong exogenous variables. More specifically, I plot three set of possible ordered pairs of delinquency and loan given unconditional moments related to restrictions (9) and (10), given a range of values for finance rate from 0% to 5%. The difference among each set is only the endowment level which ranges from USS$ 35 to US$ 52 thousands of 2017:4 dollars, minimum and maximum values respectively during the period from 1987:1 to 2018:1. Formally, the locus is given by the following equations

$$\mathbb{E}[(1+r_t).(1-\boldsymbol{d}_{t+1})] = \frac{1-cov\left[e^{-\delta\frac{u'(C_{t+1})}{u'(C_t)}},\ (1+r_t).(1-\boldsymbol{d}_{t+1})\right]}{\mathbb{E}\left[e^{-\delta\frac{u'(C_{t+1})}{u'(C_t)}}\right]} \tag{14}$$

$$\mathbb{E}[(b_t.(1+r_t)] = \lambda.\mathbb{E}(\boldsymbol{e}_{t+j}) - \frac{cov[e^{-\delta}u'(\boldsymbol{C}_{t+1})\ ,b_t.(1+r_t)-\lambda.\boldsymbol{e}_{t+j}]}{\mathbb{E}[e^{-\delta}u'(\boldsymbol{C}_{t+1})]} \tag{15}$$

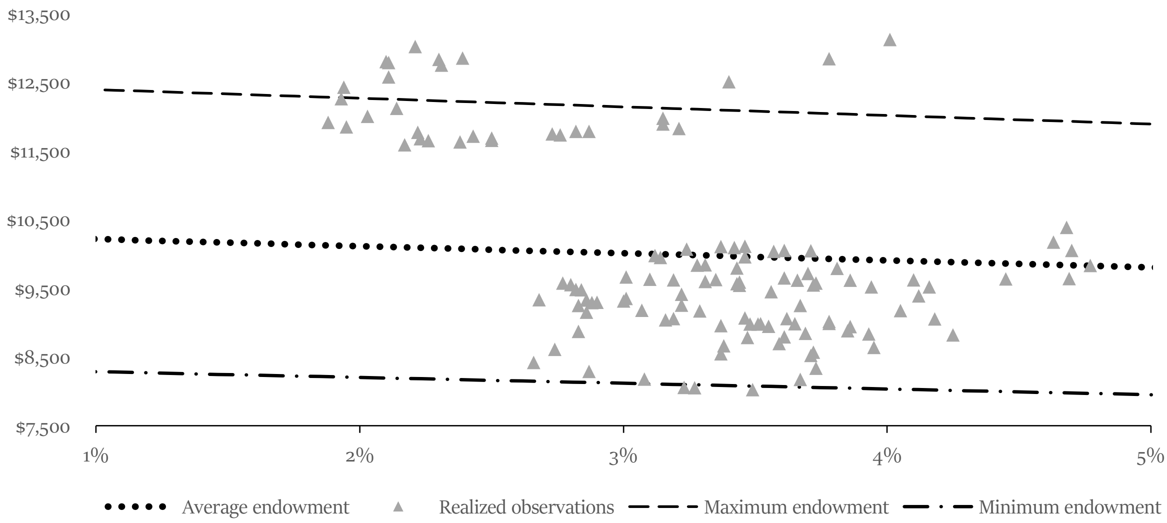


**Figure II. Delinquency and Loan.** This figure presents the personal loan delinquency (horizontal axis) and volume in thousands of 2017:4 dollars (vertical axis) dispersion for U.S. over the period from 1987:1 to 2018:1. It also presents credit bounds based on unconditional moments of first-order conditions (9) and (10).

This approach is useful to draw bounds for the household decisions on loan and delinquency and then better understanding the dispersion of such endogenous variables. It suggests smoother household credit cycles, mainly in scenarios with delinquency rate lower than 2.5%, and high level of loans. Moreover, this framework enables me to follow Lucas' (1990) rule of thumb, but now to relates $b_t$ in log with a preference's parameters, endogenous and exogenous variables via

$$ln(b_t) = -r_t + l\,n(\lambda) + ln\left(\boldsymbol{C}_{t+1}^{-\gamma}.\boldsymbol{e}_{t+j}\right) - ln(\boldsymbol{C}_{t+1}^{-\gamma}) \tag{16}$$

Based on the average values of variables during the period from 1987:1 to 2018:1, for a range of reasonable values for risk aversion between 0 and 4, the garnishment parameter $\lambda$ may range from 0.2355 and 0.2467. These limits are aligned to the values I find in all exercises performed here.

## V. Concluding Remarks

In the quest towards the partial or total resolution of famous and robust financial markets stylized facts, i.e. puzzles, macro-finance literature has explored a wide range of alternative techniques, data, preferences and market structures. I enter this debate by revisiting the most primitive choices of the representative agent and incorporating household debt and delinquency decisions into a standard quantitative model of lifecycle consumption-saving-investment.

This theoretical innovation does not compete with or rival the extensive literature mentioned above. On the contrary, I invite theoretical and empirical researchers who have proposed or applied such theories to revisit macro-finance puzzles by assuming this extended system of Euler equations.

Theoretically, the key is the possibility of issuing a debt and in the next period skip or not this obligation. It works as an additional, rare and versatile asset relevant in terms of market completeness due to its endogenous payoff. In practice, I am imposing that the model has to account for this asset, in addition to the assets that one wishes to price. I claim that the satisfactory empirical findings reported here come from this spam of contingent claims, rather than higher variance of the stochastic discount factor, which has been usually the purpose in many other approaches.

Any advance in the challenge of finding quantitative and theoretically explicit measures of main preference parameters that account for asset pricing facts is far from trivial. More specifically and aligned to my contribution, adding a security to span contingent claims is not necessarily relevant. Krueger and Lustig (2010) provide general conditions under which opening or closing down insurance markets for idiosyncratic risks has no effect on asset pricing. In short, the Equity Premium Puzzle applies to a much larger class of incomplete markets models.

In terms of future research agenda, I suggest revisiting other domestic market anomalies, and given the high volatility of credit and the levels of delinquency in emerging economies, this approach can be useful to price correctly the equity premium in these countries. Finally, da Costa et al. (2015) and Matos and da Costa (2016) argue that progresses on preference-based solutions to puzzles in domestic financial markets will also deal with puzzles in the foreign exchange market. The framework proposed here is supposed to provide researchers to combine it with any among all these research promising routes to account for international finance puzzles.

## Compliance with Ethical Standards

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Ethical approval: This paper does not contain any studies with human participants or animals performed by any of the authors.

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I add to asset pricing theory by incorporating household debt and delinquency decisions into a standard quantitative model of lifecycle consumption-saving-investment. Deriving this extended system of Euler equations does not alter consumption-based fundamental asset pricing equation. It imposes that the pricing kernel has also to account jointly for two new first-order conditions. I perform empirical exercises aiming to account for equity premium in U.S. and to price six Fama-French portfolios. I find significant elasticity ranging from 0.15 to 0.75 and risk aversion from 1.33 to 3.51. These investor decisions may be playing a relevant role in terms of completing markets.

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